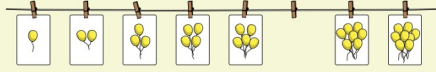
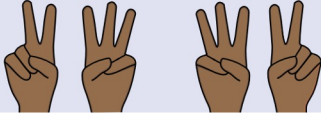
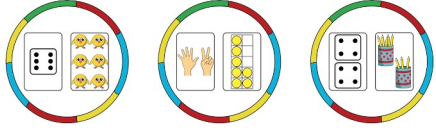
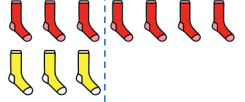
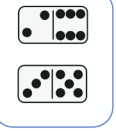
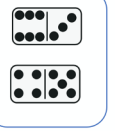
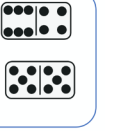
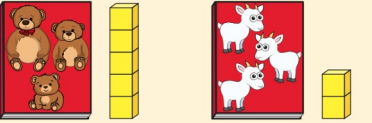
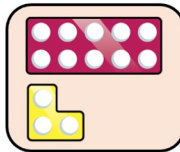
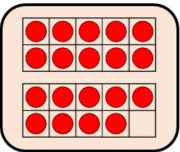
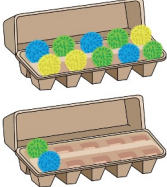
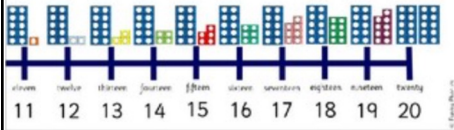
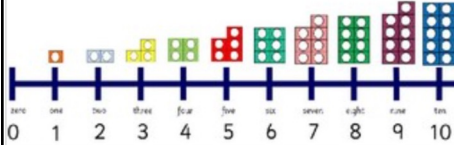
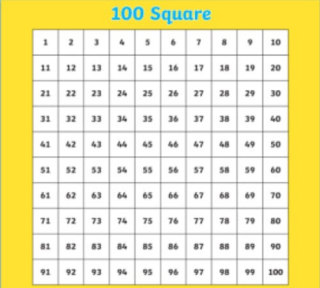
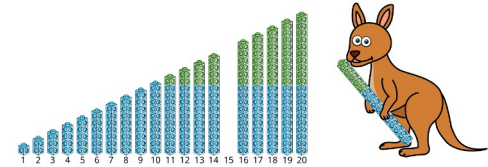


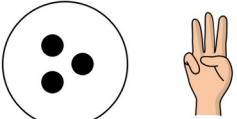
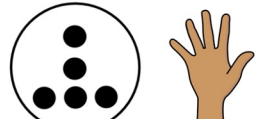
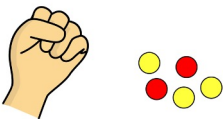
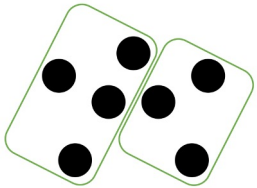
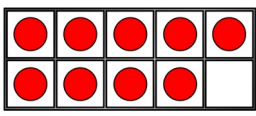
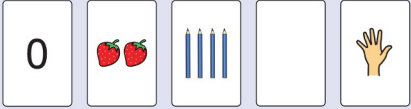
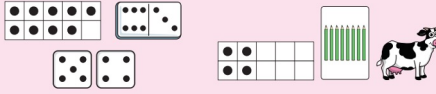
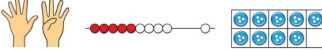
Reception - Comparison

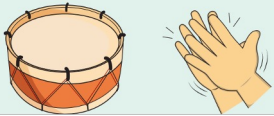
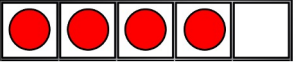
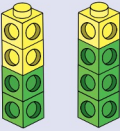
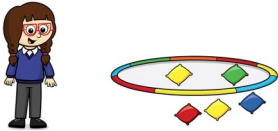
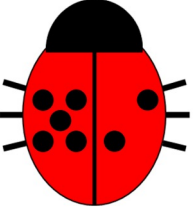
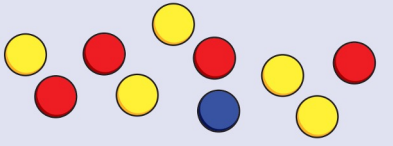
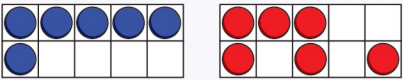
Development Matters Guidance	When taught?	
Count objects, actions and sounds The Counting Principles The one-to one principle – children assign one number name to each object that is being counted. Children need to ensure they count each object only once, ensuring they have counted every object. The stable order principle – children understand that, when counting, the numbers have to be said in a certain order The cardinal principle – children understand that the number name assigned to the final object in a group is the total number of objects in that group The abstraction principle – understanding that anything can be counted, including things that cannot be touched e.g. sounds The order-irrelevance principle – understanding that the order in which we count a group of objects is irrelevant, there will still be the same number. See White Rose Scheme of learning for how to teach these and common misconceptions	Autumn 1 & 5 Spring 1, 3, 4 & 5 Summer 1 & 6	Take children on a number hunt.  Where can they find 1, 2 and 3? Do they count to find how many? Allow time to sort the different objects into different groups based on their quantity. Encourage children to create their own collections of 1, 2 and 3 Place birthday cards representing 1–8 on a washing line. Without showing children, remove one of the cards.  Ask children to tell you which card is missing and explain how they know. This can be extended to removing two cards. Prompt children to count to 5 on their fingers and count back from 5 Encourage children to show 4 or 5 using their fingers. Is there more than one way? Is it possible to show 4 or 5 using two hands rather than just one?   Give the children three hoops to represent 6, 7 and 8 and ask children to sort the cards into the correct hoop.
Compare numbers		 There are more <u>red</u> socks. There are fewer <u>yellow</u> socks.  There are more <u>beanbags</u>. There are fewer <u>footballs</u>. fewer than  9 spots  more than  Ask children questions to make comparisons for a purpose. Set up a voting station for them to vote for their favourite book. Display two books and ask children to place a cube next to their favourite. Compare the number of votes by building towers using the cubes. Which is the most popular book?   Prompt children to see how many buttons they can hold in one hand and compare with their partner. Who has more? Who has fewer? Who has the same?

Development Matters Guidance	When taught?	
Count beyond 10	Summer 1 & 6	 I can see <u>12</u>    Ask them to order the towers to identify which one is missing. How do they know it? Can they build it? From Development Matters: Count verbally beyond 20, pausing at each multiple of 10 to draw out the structure, for instance when playing hide and seek, or to time children getting ready. Provide images such as number tracks, calendars and hundred squares indoors and out, including painted on the ground, so children become familiar with two-digit numbers and can start to spot patterns within them

Reception - Counting

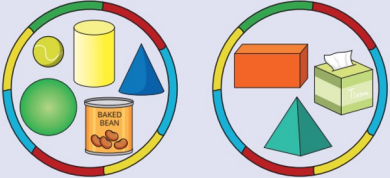
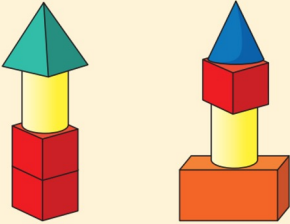
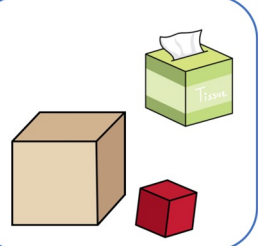
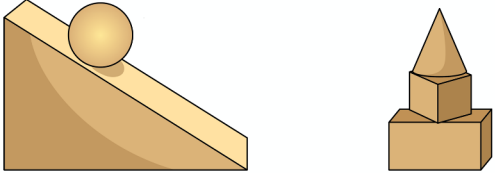
Reception - Cardinality

Development Matters Guidance	When taught?	
Subitise	Autumn 3 & 5 Spring 1, 3 & 5 Summer 6	How many dots?  Show the number on your fingers. How many dots?  Show the number on your fingers. Conceptual Subitising  I can see <u>2</u> red and <u>3</u> yellow. There are <u>5</u> altogether.  <u>4</u> is a part and <u>3</u> is a part. The whole is <u>7</u> Provide children with a range of dot plates to 10  Ask children to talk about what they see and how they see it. Encourage them to talk about the groups they can see within the whole.
Link the number symbol (numeral) with its cardinal number value		We need to set the table. How many cups do we need?  There are <u>3</u> cups altogether. Prepare a bag containing the numerals 0 to 5 As you pull out a numeral, give children a task to do to represent that number. For example, if you pull out a 2, the children could take two giant strides, do two claps or find two pebbles and bring them back.  9  There are <u>9</u> counters. Give each child a picture card. Prompt children to find a partner with the same number or amount as themselves.  Encourage children to see if there are more than two cards that match. How does the card represent the number? Provide children with a range of representations or picture cards showing 9 and not 9 Prompt children to sort the different representations into groups that show '9' and 'not 9'  Repeat this activity with representations of 10 Prompt children to represent 9 and 10 on their fingers. What do they notice?  Encourage them to show 9 and 10 in a range of different ways, such as with a bead string, with cubes or with buttons on a ten frame.

Development Matters Guidance	When taught?	
Understand the 'one more than/one less than' relationship between consecutive numbers	Autumn 3 & 5 Spring 1, 3 & 5 Summer 2, 4 & 6	Drum with the children and get them to copy your beats. Once children can copy, ask them to do 1 more beat than you with 1, 2 or 3 beats.  Model the rhyme <i>Three Little Speckled Frogs</i> with the children. Use both children and props at the front of the class to emphasise the 1 less pattern and the amount decreasing. Children can then build towers with cubes to represent the frogs and when they are jumping into the pool.  and objects to count with. Give children a number to make on their five frame.  1 2 3 4 5 Prompt children to point to the number they have made on the number track. Encourage them to show 1 more. How many are there now?   1 less than <u>4</u> is <u>3</u> Some children may be able to be 'stepped up' by using a number line to find one more/one less.
Explore the composition of numbers to 10 <i>(Children will use five frames and then tens frames later in the year to support this)</i>		Ask children to build different towers of 4 or 5 using both colours. Encourage children to compare the towers and talk about the parts that they can see.  <u>2</u> is a part and <u>3</u> is a part. The whole is <u>5</u>  What do you see? How do you see it?  <u>5</u> is a part and <u>2</u> is a part. The whole is <u>7</u>  Can you find another way?
Automatically recall number bonds for numbers 0-5 and some to 10		Put together a collection of items in a feely bag that represent numbers 0 to 5  Take each item out and discuss what amount is represented by each object. Ask children to show this amount on their fingers or on a five frame with counters. Provide children with sprayed butter beans or counters in three different colours. Ask them to count out 10 and shake them in their hand before dropping them. How many are red? How many are yellow? How many are blue?  Provide each child with a number shape. Encourage them to make a bond to 10 with a partner.  Which number shapes combine to make a total of 10? Children can check by placing their two number shapes on top of the number 10 piece. Compare the different tens that are made. Model representing numbers to 10 on a ten frame in different ways and talk about what the children notice.  What does each arrangement tell us about that number?

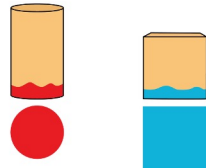
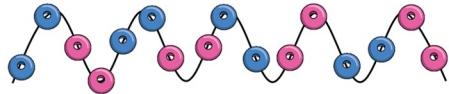
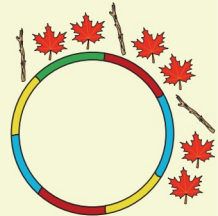
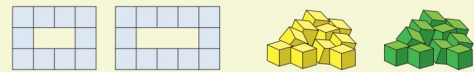
Reception – Spatial

Awareness

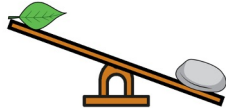
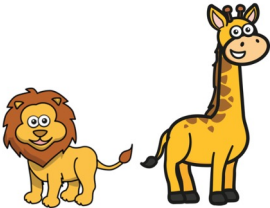
Development Matters Guidance	When taught?	
Select, rotate and manipulate shapes in order to develop spatial reasoning skills	Spring 6 Summer 3	Provide children with a range of 3-D shapes and real objects. Encourage them to sort the shapes into groups within hoops. Prompt children to talk about why they have sorted the shapes that way. Is there another way we could sort them?  After reading traditional tales such as <i>Rapunzel</i>, children explore building towers. Which shapes do they need to use to build Rapunzel's tower? Which shapes do they need to place at the bottom of the tower? Which shapes do they need to place at the top? Prompt them to say why they have chosen to place that shape in that position.  spheres   cubes   Provide children with different 3-D shapes and a ramp. Explore which 3-D shapes roll down the ramp and which do not.  What do they notice about the shapes that do roll? What makes a shape good for stacking?

Reception – Shape and

Pattern

Development Matters Guidance	When taught?	
Compose and decompose shapes so that children recognise a shape can have other shapes <i>within</i> it, just as numbers can	Autumn 6 Spring 6 Summer 3	Have a range of flat paper squares and rectangles for children to explore. Ask children to investigate which new shapes they can make by combining different combinations of the shapes.  Task children to make a large, medium or small square or rectangle. Is there a different way to make the same size shape? Encourage children to explore printing with 3-D shapes using paint. What do they notice?  What 2-D shapes can they see? Ask children to predict what footprint a shape will make.
Continue, copy and create repeating patterns	Autumn 2 Spring 6 Summer 5	What pattern can you see?  What will come next? Ask children to arrange patterns around a circle, such as a hoop or a paper plate. Prompt them to consider how they will continue the pattern all the way round. Does their pattern fit? Encourage children to view other patterns in the environment and replicate them.  Provide frames with a set number of spaces and a range of loose parts. Ask children to build patterns around the frame by putting one item in each space.  Prompt them to investigate whether AB, ABC, ABB, AAB, AABB and ABBA patterns will fit around the frame. Which patterns will fit exactly, and which will not?

Reception – Measure

Development Matters Guidance	When taught?	
Compare length, weight and capacity <i>Using real world objects in the provision</i>	Spring 2 & 4 Summer 6	 The <u>apple</u> is heavier than the <u>strawberry</u>.  The heavier object is <u>lower</u> on the balance scale. Provide a selection of containers of different shapes and sizes and ask children to investigate which holds the most water.  Place the containers in order from the smallest to the greatest capacity.  The <u>blue</u> ribbon is the shortest.  The ribbons are <u>the same length</u>. Provide children with ribbons of different lengths, widths and colours.  Prompt them to line up the lengths of ribbon in order from longest to shortest.  The <u>giraffe</u> is taller than the <u>lion</u>. Encourage children to use simple non-standard units to measure the heights of different objects. For example, use paperclips strung together to measure the height of a pot. 

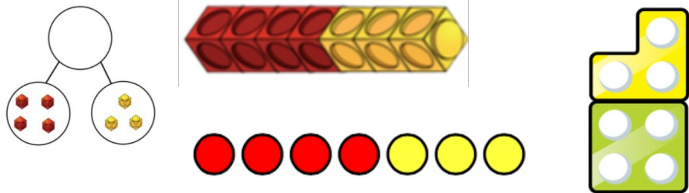
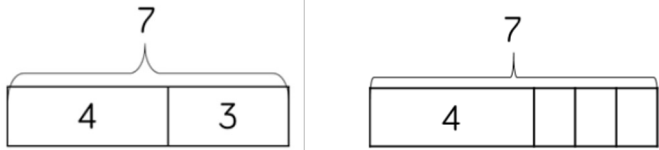
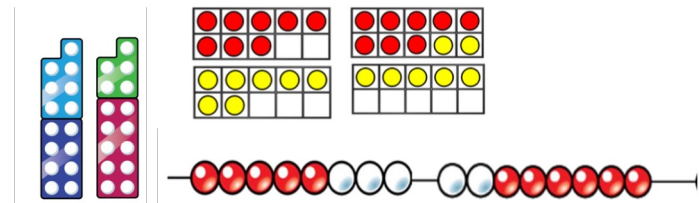
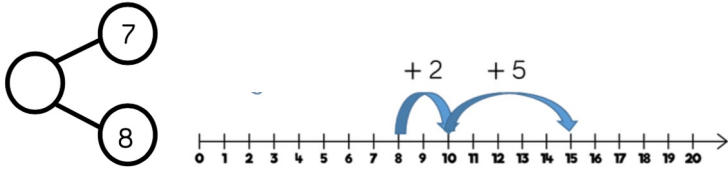
End of Year 1 Fluency Milestones

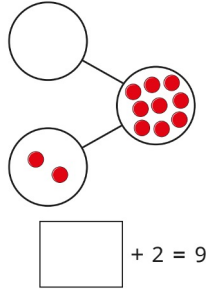
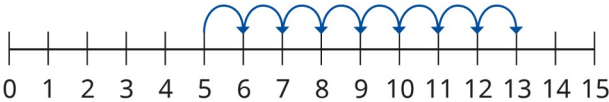
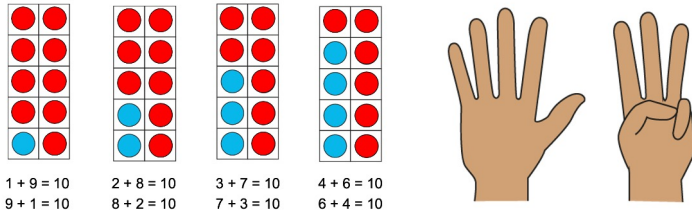
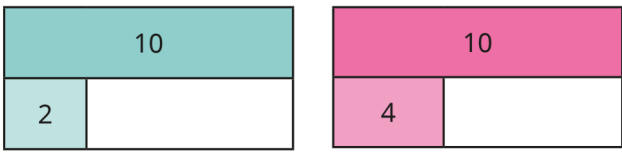
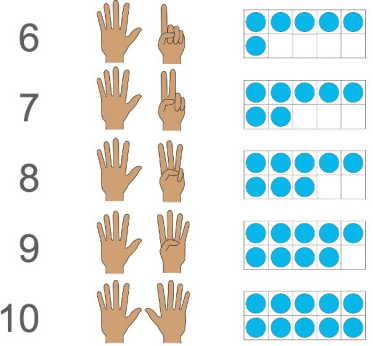
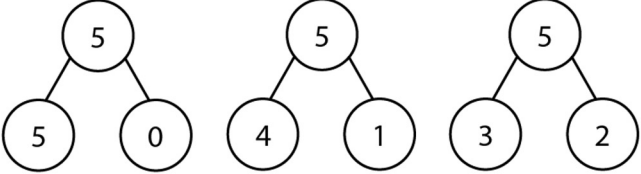
Procedural Fluency

- Use number lines efficiently to count on and back
- Add and subtract within 10 – moving beyond counting forwards or backwards in ones, with automatic recall (66 facts shown on the grid)
- Apply commutativity e.g., $2+3$ is the same as $3+2$
- Automatic recall of corresponding division facts e.g., $5-3$ and $5-2$

Conceptual fluency

- Know number bonds within and to 10
- Know one and two more/less than any 1-digit number
- Count forward and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning with any multiple
- Count forwards and backwards through the odd numbers
- Fact families for some numbers within 20

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial
Add two one digit numbers within and to 10	Autumn Block 2 Spring Block 2		
Add one and two digit numbers to 20, including zero			

Solve one step problems that involve addition using pictorial representations, and missing number problems	Autumn Block 2 Spring Block 2		a) $5 + \square = 13$ 
1NF-1 Develop fluency in addition facts within and to 10	Autumn Block 2 (Steps 5, 6, 7) Spring Block 2 (Steps 2 and 6)	 5 and 3 fingers are 8.	
1AS-1 Compose numbers to 10 from 2 parts, and partition numbers to 10 into parts			

Year 1 - Addition

Year 1 - Addition

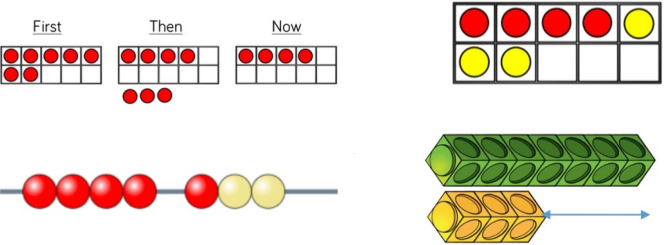
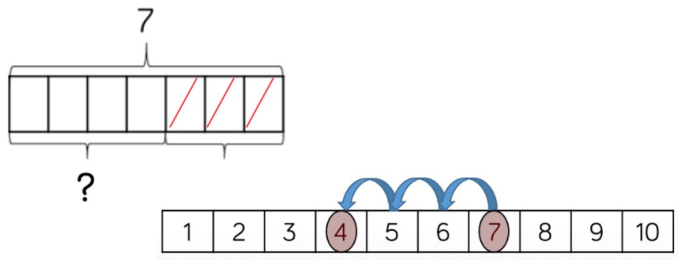
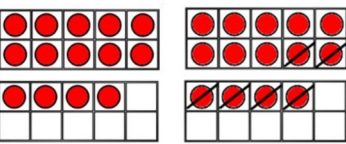
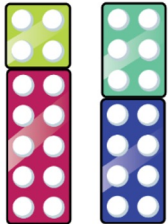
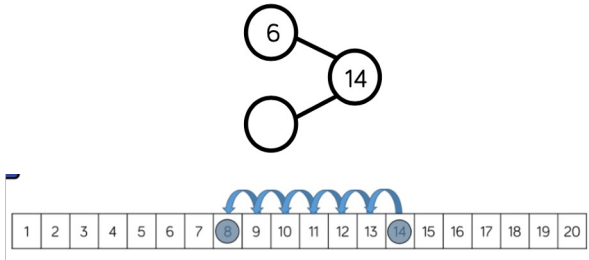
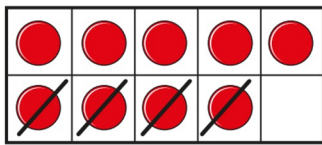
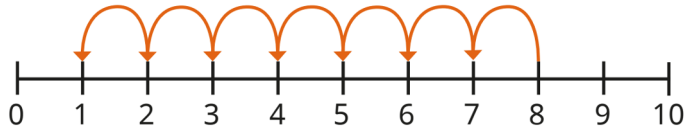
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1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8		
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7			
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6				
5	5+0	5+1	5+2	5+3	5+4	5+5					
6	6+0	6+1	6+2	6+3	6+4						
7	7+0	7+1	7+2	7+3							
8	8+0	8+1	8+2								
9	9+0	9+1									
10	10+0										

Year 1

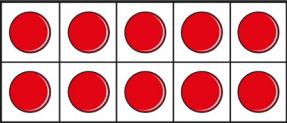
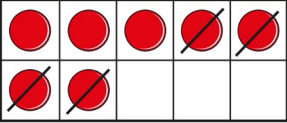
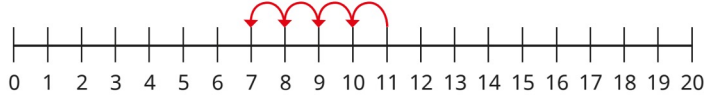
Addition Facts –

Children should be conceptually fluent by the end of Year One.

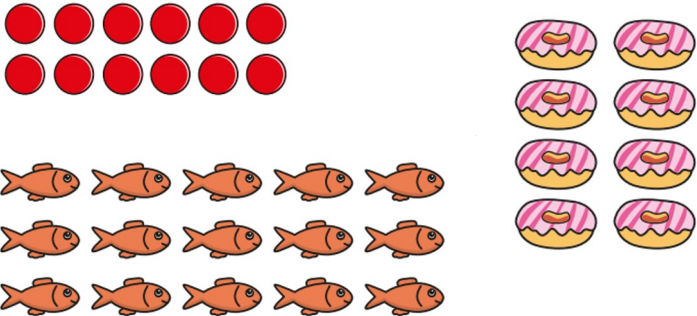
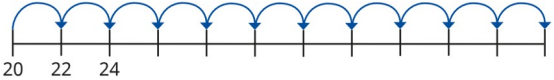
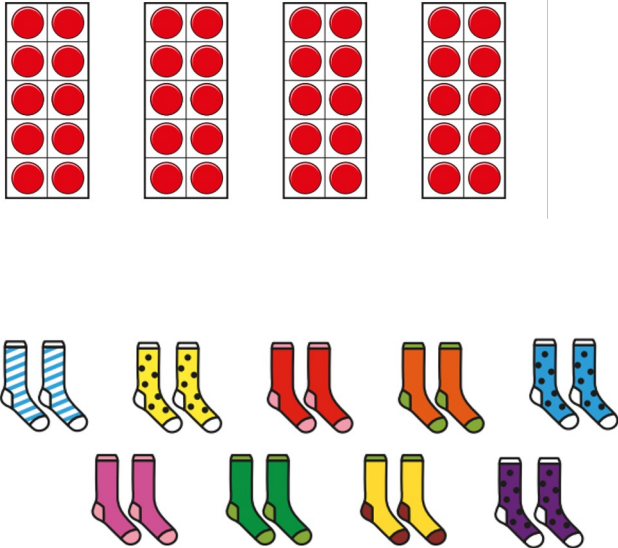
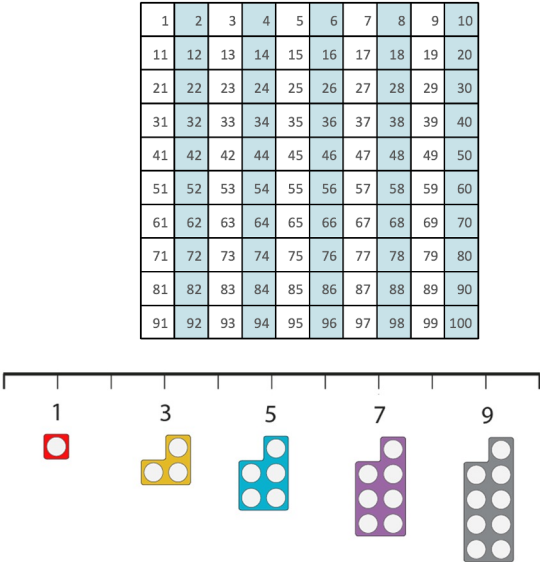
Year 1 – Subtraction

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial
Subtract one-digit numbers within 10	Autumn Block 2		
Subtract one-digit and two digit numbers to 20, including zero	Autumn Block 2 Spring Block 2	 	
Solve one-step problems that involve subtraction using concrete objects and pictorial representations, and missing number problems	Autumn Block 2 Spring Block 2	Ben  Jo  	 $\square - \square = \square$

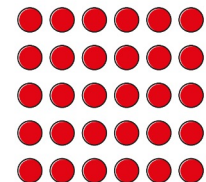
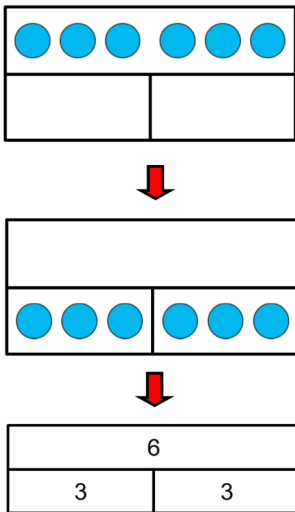
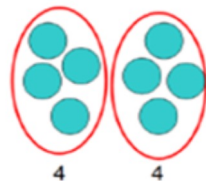
Year 1 – Subtraction

1NF-1 Develop fluency in subtraction facts within 10	Autumn Block 2 (Steps 5, 6, 7) Spring Block 2 (Steps 2 and 6)	   	a) $11 - 4 =$ 
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Year 1 – Multiplication

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial
Solve one step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with support	Summer Block 1		
1NF-2 Count forwards and backwards in multiples of 2, 5 and 10, up to 10 multiples, beginning at any multiple, and count forwards and backwards through the odd numbers	Summer Block 1 (Steps 1, 2 & 3) Summer Block 4 (Step 2) Summer Block 5 (Step 4)		

Year 1 – Division

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial
Solve one –step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with support	Summer Block 1	Arrays will be made practically using counters and other objects Ron needs to share 20 bananas between 5 boxes.  How many bananas will there be in each box? Use 30 counters.  a) Share the counters between 2 people. How many counters does each person get?	Children can begin to draw these representations   8 shared between 2 is 4 half of 6 = 3 double 3 = 6

Year 2 - Addition

Note:

In Year 2, children use **Base 10** when building numbers in place value columns as this gives them a better understanding of the size of the numbers and 'how many' ones there are in one ten to be able to understand exchanging.

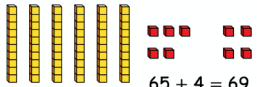
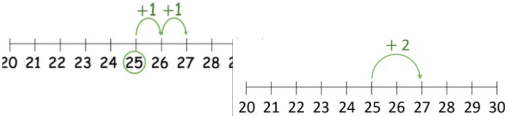
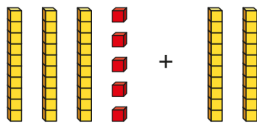
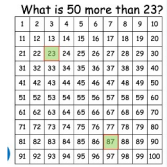
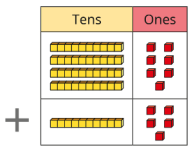
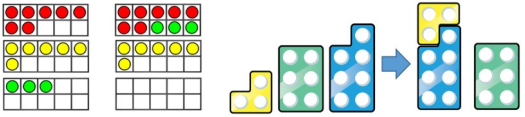
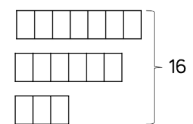
End of Year 2 Fluency Milestones

Procedural Fluency

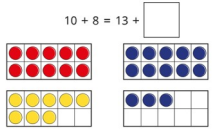
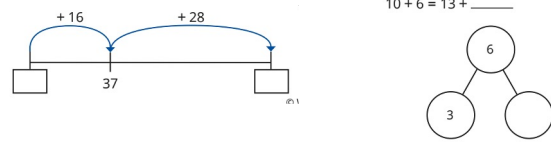
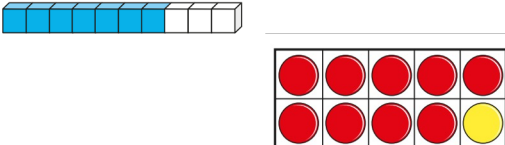
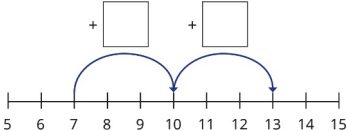
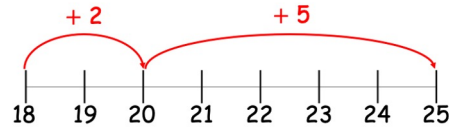
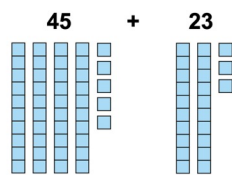
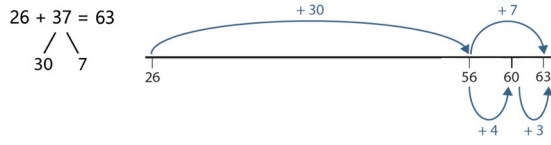
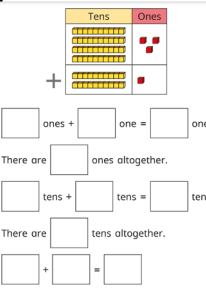
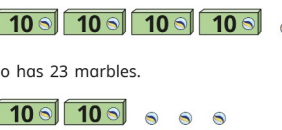
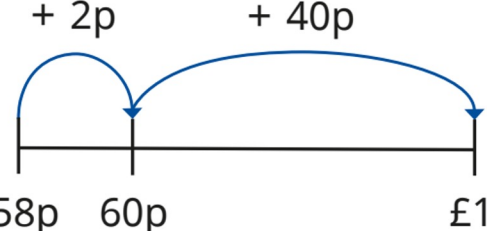
- Add by making 10 (done practically using tens frames/number lines) – this will move towards a mental strategy.
- Subtract crossing 10 using partitioning (using concrete manipulatives or pictorially) – this will move towards a mental strategy.
- Add 2-digits and 1 digit using partitioning and 'counting on' using tens frames and number lines. Children will end the year being able to use this as a mental strategy
- Subtract a 1-digit number from a 2-digit number using partitioning to make and cross ten using tens frames and number lines. Children will end the year being able to use this as a mental strategy.

Conceptual fluency

- Know addition and subtraction bonds to 20
- Know number bonds to 100
- Know doubles to 20
- Know 'near doubles' e.g., 5+6
- Rapid recall of multiplication and division facts for 2, 5 and 10 multiplication tables

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Add numbers using concrete objects, pictorial representations and mentally including				
> a 2-digit number and ones	Autumn Block 2	 $65 + 4 = 69$		$25 + 2 = 27$
> a 2-digit number and tens				$24 + 20 = \square$
> two 2-digit numbers			As concrete but children draw their own place value charts with Base 10.	$16 + 47 = \square$
> three 1-digit numbers				$7 + 6 + 3 = 16$

Year 2 - Addition

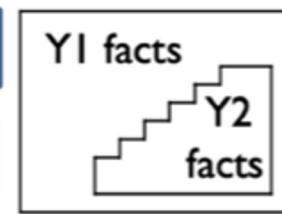
National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Solve addition problems using objects and representations	Autumn Block 2	 10 + 8 = 13 + Ron has these grapes.  Teddy has 25 more grapes than Ron. How many grapes does Teddy have?	 10 + 6 = 13 +	$10 + 8 = 14 + \square$
2NF-1 Secure fluency in addition facts within 10	Autumn Block 2 (steps 1, 6, 8, 11)			$7 + 3 = \square$ c) $5 + \square = 10$ $1 + 9 = \square$ d) $6 + \square = 10$
2AS-1 Add across 10	Autumn Block 2 (Steps 9-12)			$15 + 9 =$ + =
2AS-3 Add within 100. by applying related 1-digit addition facts: + only ones or only tens to a 2-digit number	Autumn Block 2 (Steps 14)	 45 + 23 = 40 + 20 = 60 5 + 3 = 8 60 + 8 = 68	 26 + 37 = 63	$25 + 2 = 27$ $24 + 20 = \square$
2AS-4 Add within 100 by applying related 1-digit addition facts: add any 2-digit numbers)	Autumn Block 2 (15-19) Spring Block 1 (8 & 9) Spring Block 3 (5)	 Ron has 42 marbles.  Jo has 23 marbles. How many marbles are there altogether?	 58p 60p £1	$37 + 14 = \square$ $22 + 19 = \square$

Adding 1

Bonds to 10

Adding 10

Bridging/
compensating



Adding 2

Adding 0

Doubles

Near doubles

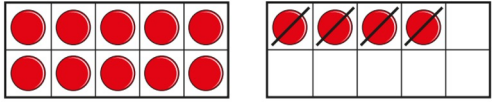
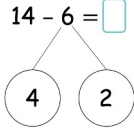
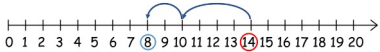
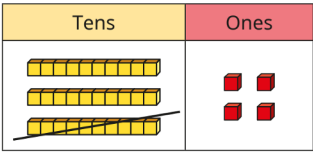
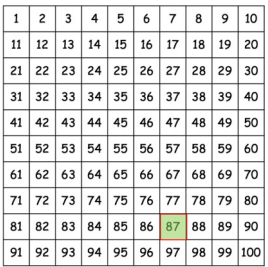
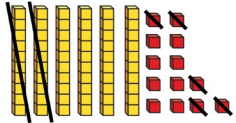
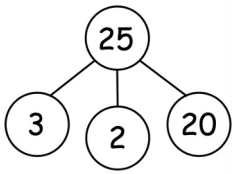
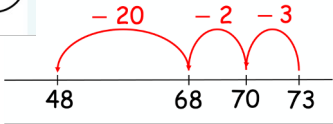
Key Stage 1

Addition Facts –

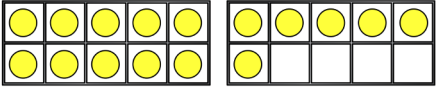
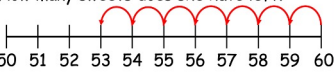
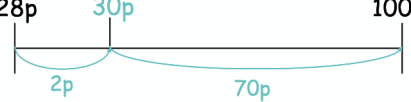
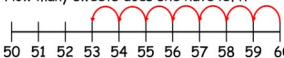
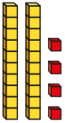
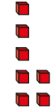
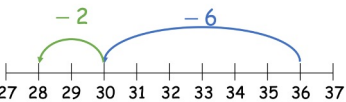
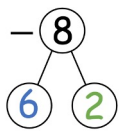
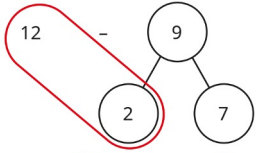
Children should be conceptually fluent by the end of Year Two.

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0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1	1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

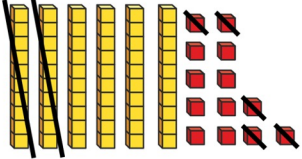
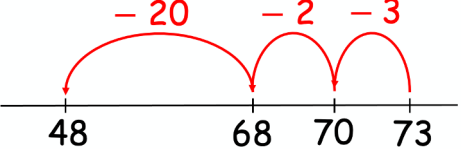
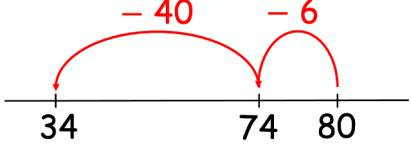
Year 2 - Subtraction

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Subtract numbers using concrete objects, pictorial representations and mentally including				
> a 2-digit number and ones	Autumn Block 2	 $14 - 4 = 10$	$14 - 6 = \square$  $14 - 4 = 10$ $10 - 2 = 8$ I can partition 6 into <u>4</u> and <u>2</u> 	$11 - 5 = \square$ $) 14 - 6 = \square$
> a 2-digit number and tens		 $\square - \square = \square$	$87 - 30 =$ 	$34 - 20 = \square$ $57 - 20 = \square$
> two 2-digit numbers		 $1 \text{ ten} = \underline{10} \text{ ones}$ $\underline{13} \text{ ones} - 5 \text{ ones} = \underline{8} \text{ ones}$ $\underline{6} \text{ tens} - 2 \text{ tens} = \underline{4} \text{ tens}$	 	$48 - 11 = \square$ $27 - 16 = \square$

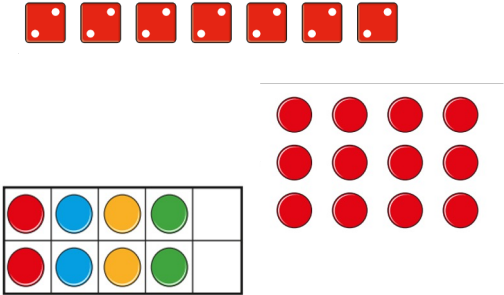
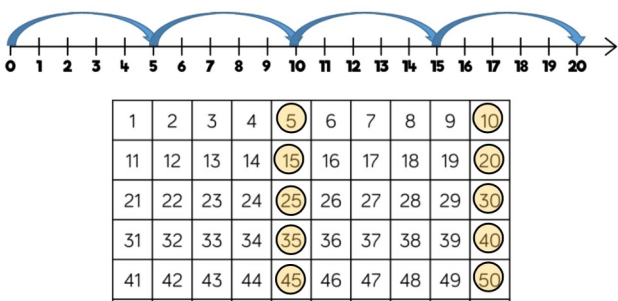
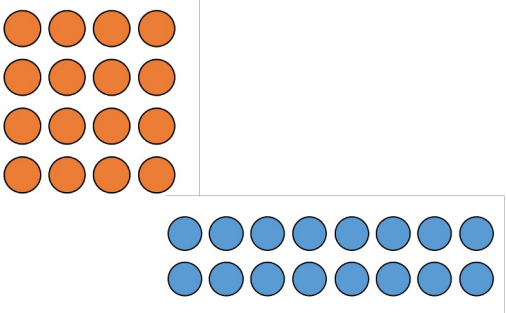
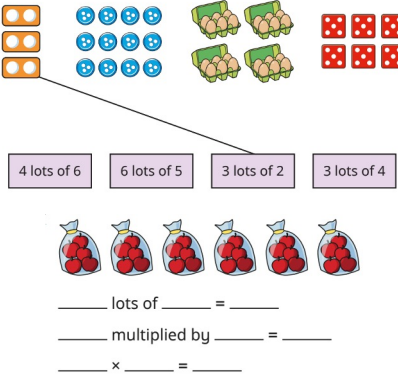
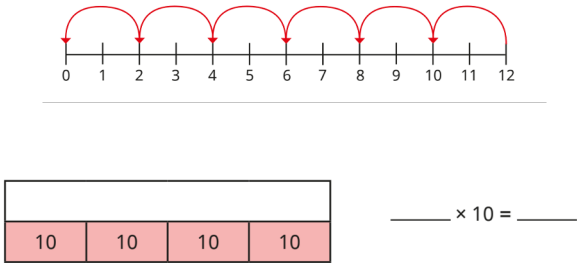
Year 2 - Subtraction

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
2AS-1 Subtract across 10	Autumn Block 2 (Steps 10, 11 & 12)	Use the ten frames to calculate $20 - 4$  $20 - 4 = 16$ Complete the sentence. If $10 - 4 = 6$, then $20 - 4 = 16$	Dora has 60 sweets.  She gives 3 to Jack and 4 to Kim. How many sweets does she have left?  If $10 - 7 = 3$, then $60 - 7 = 53$	$11 - 3 = \square$ $15 - 6 = \square$ $12 - 6 = \square$ $13 - 7 = \square$
2AS-2 Recognise the subtraction structure of 'difference' and answer questions of the form, "How many more...?"	Spring Block 1 (Step 9)	Ron has this money.  He wants to buy a yo-yo.  How much money does he have left? 28p 	28p 30p 100p  Dora has 60 sweets.  She gives 3 to Jack and 4 to Kim. How many sweets does she have left?  If $10 - 7 = 3$, then $60 - 7 = 53$	Change  Total price $3p + 97p = £1$  $4p + 96p = £1$
2AS-3 Subtract within 100 by applying related one-digit subtraction facts: subtract only ones or only tens from a 2-digit number	Autumn Block 2 (Steps 10, 11, 12 & 14)	 -  = 	Mo is calculating $36 - 8$  $36 - 8$  $36 - 6 = 30$ $30 - 2 = 28$ 	 $10 - 7 = \square$ $31 - 7 = \square$ $46 - 9 = \square$

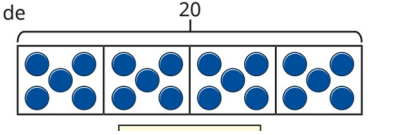
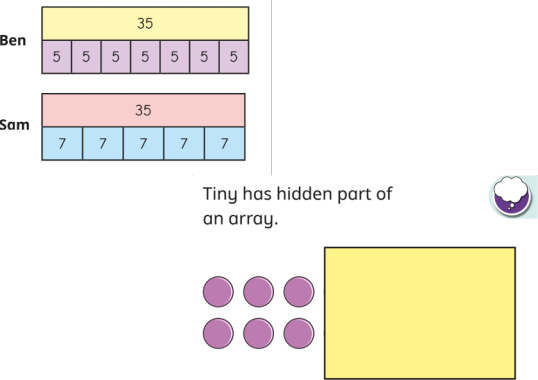
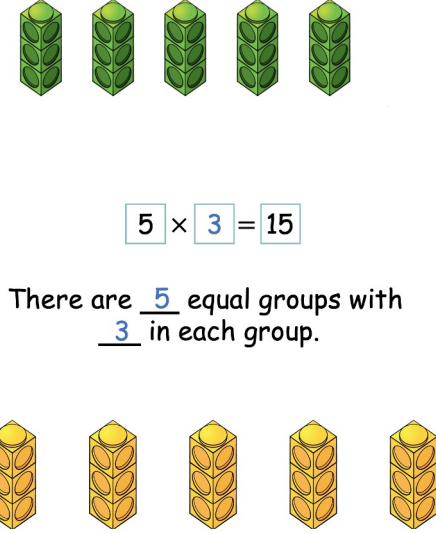
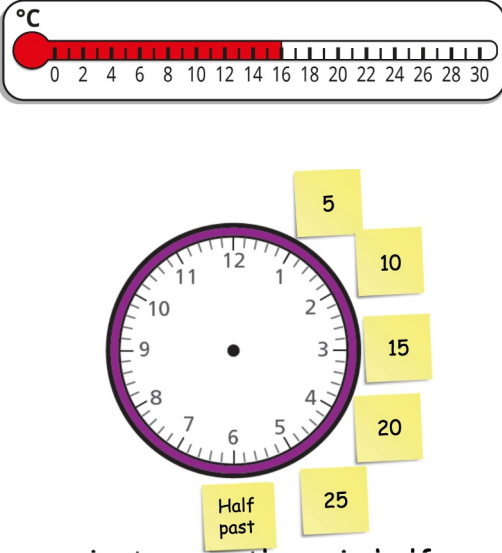
Year 2 - Subtraction

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
2AS-4 Subtract within 100 by applying related one digit subtraction facts: subtract any 2 two-digit numbers	Autumn Block 2 (Steps 17, 18 & 19) Spring Block 1 (Step 9) Spring 3 (Step 5)	$73 - 25 =$  1 ten = <u>10</u> ones <u>13</u> ones - 5 ones = <u>8</u> ones <u>6</u> tens - 2 tens = <u>4</u> tens	 	Children will use abstract number sentences to record alongside concrete and pictorial methods

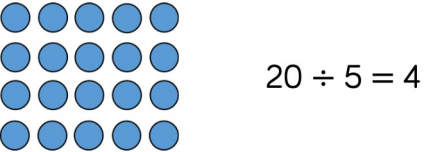
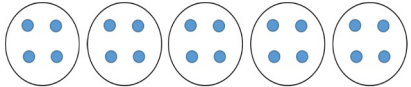
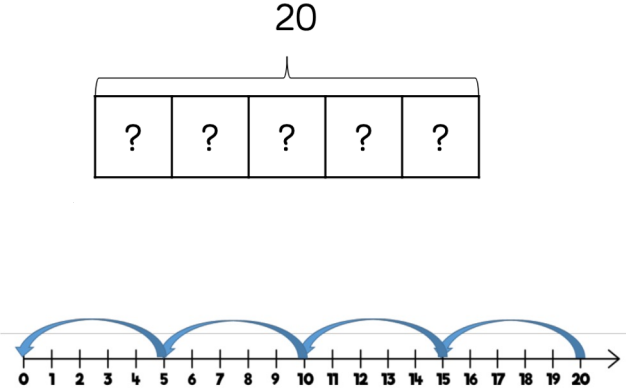
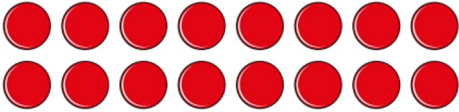
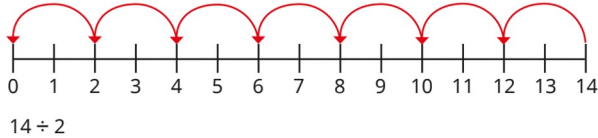
Year 2 - Multiplication

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers	Spring Block 2			$5 + 5 + 5 + 5 = 20$ $4 \times 5 = 20$ $5 \times 4 = 20$
Show that multiplication of two numbers can be done in any order (commutative)	Spring Block 2		As concrete but children draw arrays knowing equal groups, rows and columns	$6 + 6 + 6 = 18$ $3 \times 6 = 18$ $3 + 3 + 3 + 3 + 3 + 3 = 18$ $6 \times 3 = 18$
Calculate mathematical statements for multiplication within the multiplication tables and write them using the x and = signs	Spring Block 2			$5 \times 3 = \underline{\quad}$ $\underline{\quad} = 12 \times 5$ $5 \times 6 = \underline{\quad}$ $5 \times \underline{\quad} = 35$

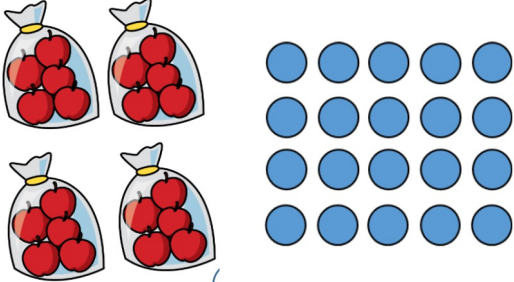
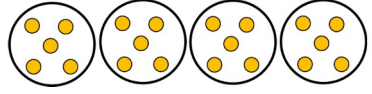
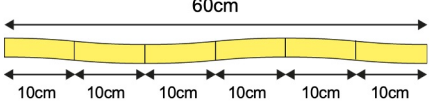
Year 2 - Multiplication

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Solve problems involving multiplication using materials, arrays, repeated addition, mental methods including problems in contexts	Spring Block 2	 Find 2×5 and 5×2	 Tiny has hidden part of an array.	One bag holds 5 apples. How many apples do 4 bags hold? $5 + 5 + 5 + 5 = 20$ $4 \times 5 = 20$ $5 \times 4 = 20$
2MD-1 Recognise repeated addition in contexts, representing them with multiplication equations and calculating the product within the 2, 5 and 10 multiplication tables	Spring Block 2 (Steps 4, 5, 9, 13, 15 & 17) Spring Block 4 (step 8) Summer Block 2 (Steps 5 & 6)	 $5 \times 3 = 15$ There are <u>5</u> equal groups with <u>3</u> in each group.		$5 + 5 + 5 = 15$ $3 \times 5 = 15$

Year 2 - Division

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Recall and use division facts for the 2, 5 and 10 multiplication tables including recognising odd and even numbers	Spring Block 2	 $20 \div 5 = 4$  $20 \div 5 = 4$		$20 \div 5 = 4$
Calculate mathematical statements for division within the multiplication tables and write them using the division and equals signs	Spring Block 2	 $____ \times 2 = 16$ $16 \div 2 = ____$		$70 \div 10 = ____$ $5 = ____ \div 10$

Year 2 - Division

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Solve problems involving division using materials, arrays, repeated subtraction, mental methods and division facts including problems in context	Spring Block 2	There are 20 apples altogether. They are put in bags of 5. How many bags are there? 	There are 20 apples altogether. They are put in bags of 5. How many bags are there?  Children will draw out – sharing or grouping to find the answers.	There are 20 apples altogether. They are put in bags of 5. How many bags are there? $20 \div 5 = 4$
2MD-2 Relate grouping problems where the number of groups is unknown to division equations (quotative division)	Spring Block 2 (Steps 2, 7, 8, 10, 14 & 16)	 $14 \div 2 = \boxed{7}$	 $60 \div 10 = \boxed{6}$	$8 \div 2 = \boxed{}$ $15 \div 5 = \boxed{}$

Year 3 - Addition

Note:
In Year 3 , children move from using Base 10 once they understand the size of the numbers, They then step up to use place value counters instead. This makes use of concrete manipulatives more efficient as pupils are now looking at numbers with 3-digits

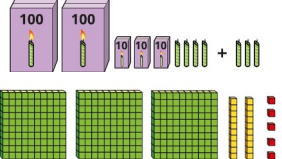
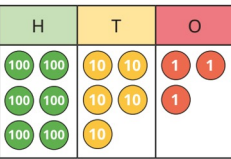
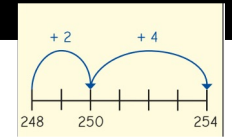
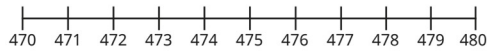
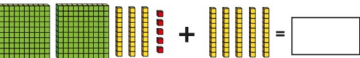
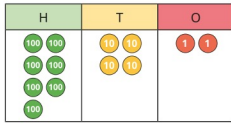
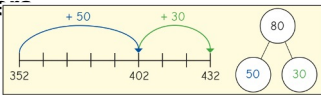
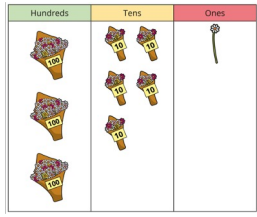
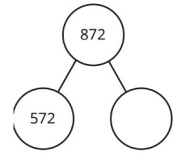
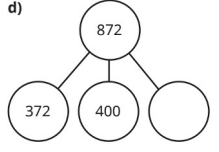
Procedural Fluency

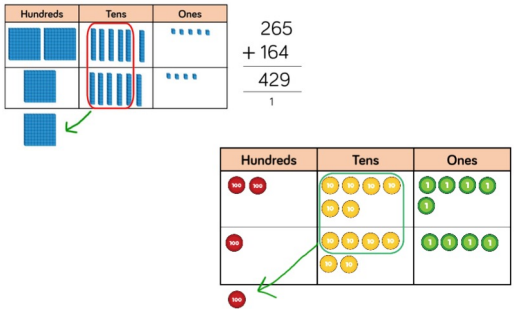
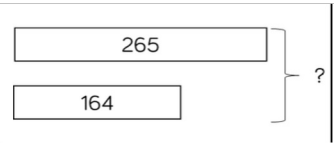
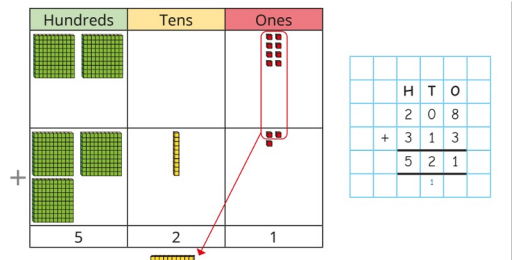
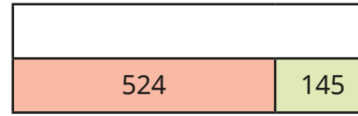
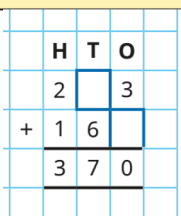
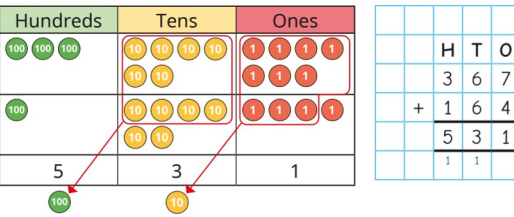
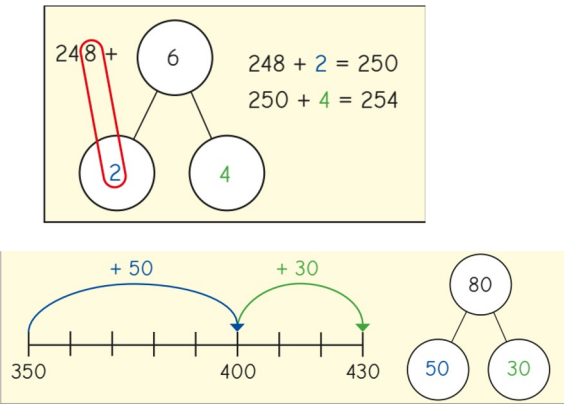
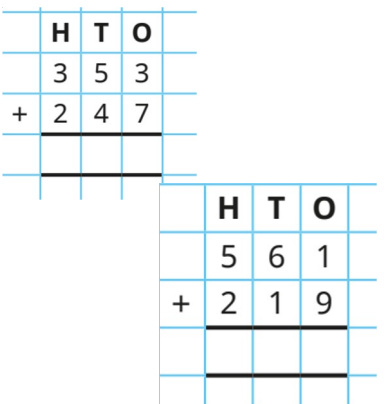
- Using nearly numbers and compensation to solve calculations. “Adding 18 is the same as adding 20 and subtracting 2”.
- Regrouping for addition and subtraction to allow bridging through multiples of ten and a hundred.
- Count on to find the difference – drawing out the use of complements to benchmark numbers e.g., 916-897. “There is 3 more to 900 and 16 more to 916.”
- Use related calculations to solve other problems. Understand that because one of the numbers in the calculation is ten times bigger, then the answer will also be ten times bigger e.g., if we know that $2 \times 6 = 12$, we also know that $2 \times 60 = 120$
- Use partitioning to multiply 2-digits by 1-digit
- Use formal column methods for addition and subtraction of two 3-digit numbers

End of Year 3 Fluency Milestones

Conceptual fluency

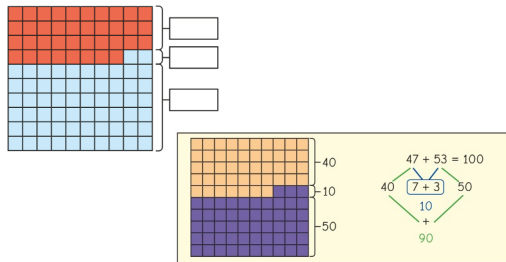
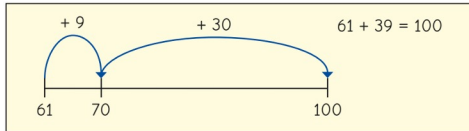
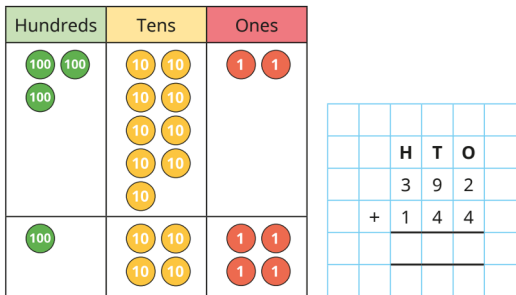
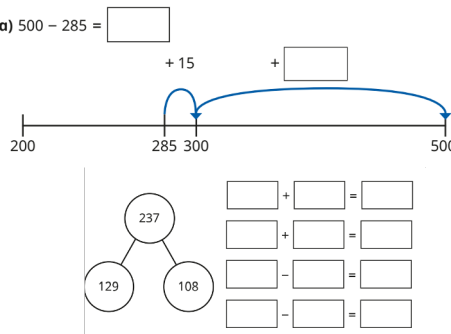
- Rapid recall of multiplication and division facts for 3, 4 and 8 multiplication tables.
- Automatic recall of addition and subtraction facts within and across 10 without counting in ones on their fingers, on a number line or in their head (see below grid) *Pupils should practise until they achieve automaticity in the mental application of these strategies.*
- Double and halve 2-digit numbers

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Add numbers mentally including				
> 3-digit number and ones	Autumn Block 2	 	 	$3 + 405 = \boxed{}$ $\boxed{} + 913 = 918$
> 3-digit number and tens		 	As concrete but children draw their own place value charts with Base 10 or Place Value counts 	$60 + 927 = \boxed{}$ $\boxed{} + 913 = 953$
> 3-digit number and hundreds			As above and also:  	$\boxed{} + 517 = 917$ $728 = \boxed{} + 328$

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Add numbers with up to 3-digits, using formal written methods of columnar addition (Including with up to 2 exchanges)	Autumn Block 2		Children also draw their own place value charts and record alongside using columnar method before moving to abstract 	$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ \hline 1 \end{array}$
Solve problems including missing number problems, using number facts, place value and more complex addition	Autumn Block 2			$263 + 60 < 319 + \square < 434 - 80$ 
3NF-1 Secure fluency in addition facts that bridge 10 through continued practice	Autumn Block 2 (steps 6, 7, 13, 14)			

Year 3 - Addition

Year 3 - Addition

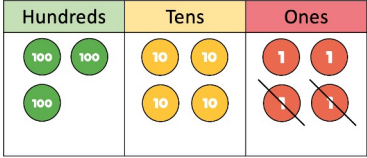
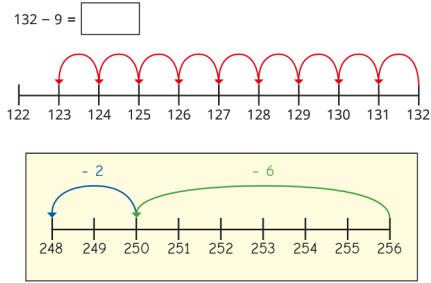
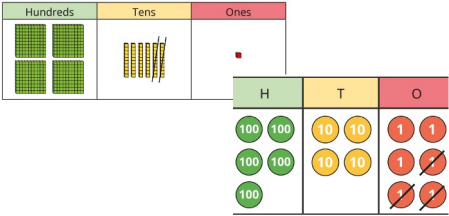
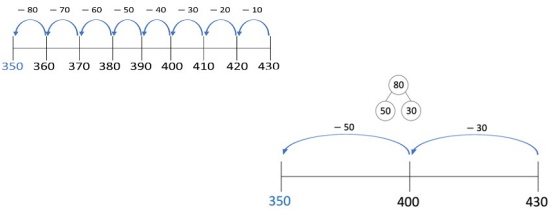
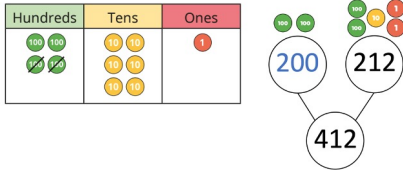
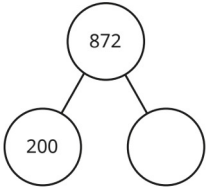
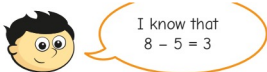
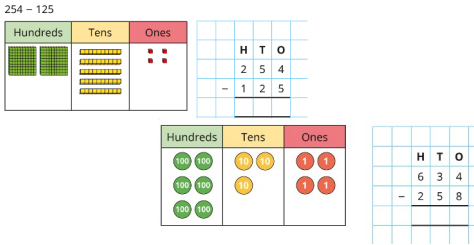
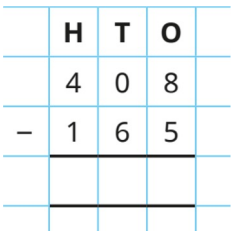
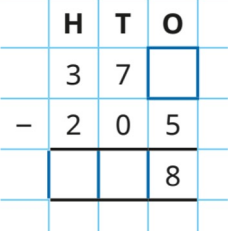
National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract																																																																
3AS-1 Calculate complements to 100	Autumn Block 2 (Step 19) Summer block 2 (Steps 4 and 5)			84 + 1__ 35 + __5 __7 + 53																																																																
3AS-2 Add up to 3-digit numbers using columnar methods	Autumn Block 2 (Steps 11, 13,14, 17)		Children draw the counters themselves and record alongside using columnar method. d) <table><tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table> <table><tr><th>H</th><th>T</th><th>O</th></tr><tr><td>3</td><td>9</td><td>2</td></tr><tr><td>+</td><td>1</td><td>4</td></tr><tr><td></td><td></td><td></td></tr></table>	Hundreds	Tens	Ones										H	T	O	3	9	2	+	1	4				<table><tr><th></th><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td>3</td><td>4</td><td>5</td></tr><tr><td>+</td><td>2</td><td>5</td><td>8</td></tr><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td></tr></table> <table><tr><th></th><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td>6</td><td>5</td><td>5</td></tr><tr><td>+</td><td></td><td>5</td><td></td></tr><tr><td></td><td>8</td><td></td><td>7</td></tr><tr><td></td><td></td><td></td><td></td></tr></table>		H	T	O		3	4	5	+	2	5	8										H	T	O		6	5	5	+		5			8		7				
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3AS-3 Manipulate the additive relationship. Understand the inverse relationship between addition and subtraction. Understand the commutative property of addition	Autumn Block 2 (Steps 21, 22) Summer Block 2 (Steps (3-5)	Continued practice of addition using place value counters and place value charts..	a) 500 – 285 =  <table><tr><td> + = </td></tr><tr><td> + = </td></tr><tr><td> - = </td></tr><tr><td> - = </td></tr></table>	+ =	+ =	- =	- =	Use formal columnar method to check subtractions using an addition <table><tr><th></th><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td>2</td><td>5</td><td>6</td></tr><tr><td>+</td><td>1</td><td>8</td><td>3</td></tr><tr><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td></tr></table>		H	T	O		2	5	6	+	1	8	3																																																
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Year 3 - Addition

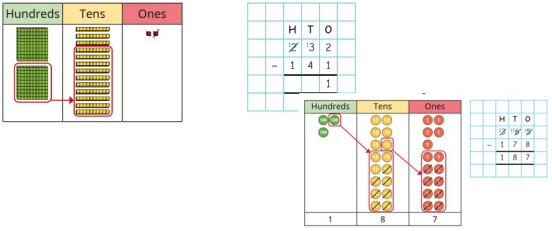
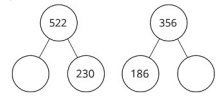
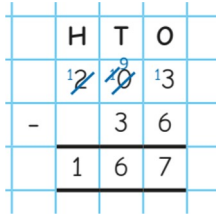
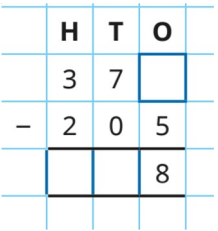
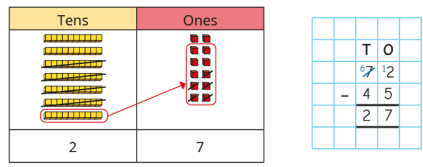
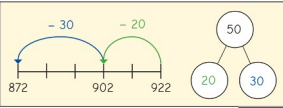
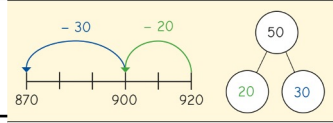
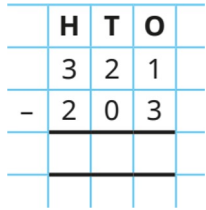
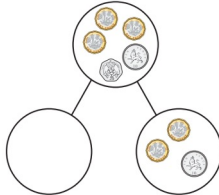
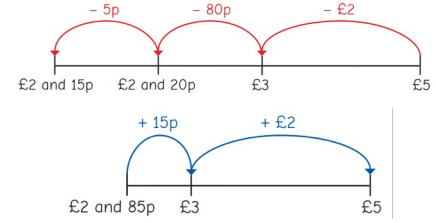
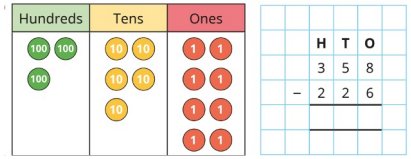
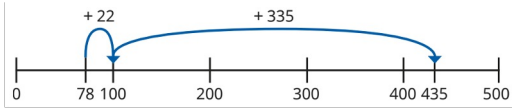
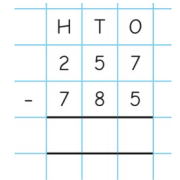
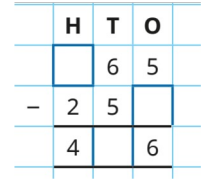
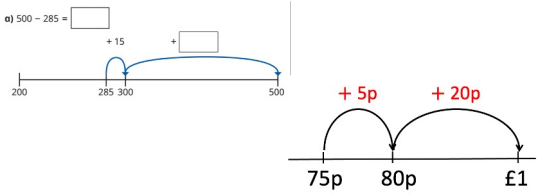
Year 3 children need to be conceptually fluent in all of these number facts to support formal methods of addition in Key Stage 2.

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1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

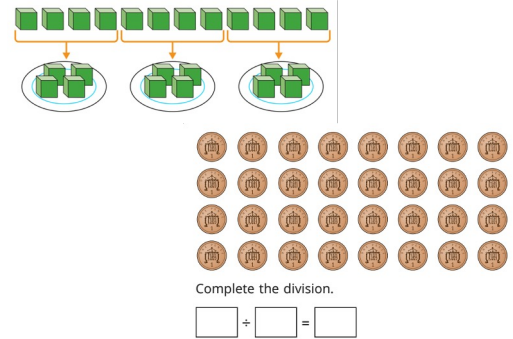
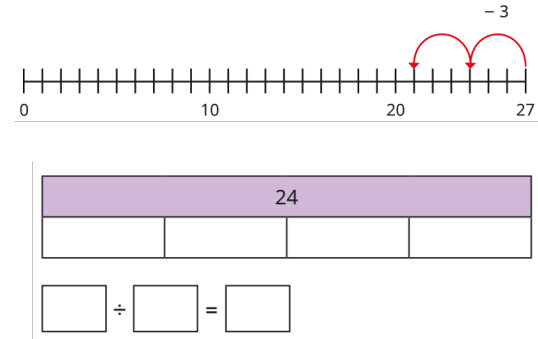
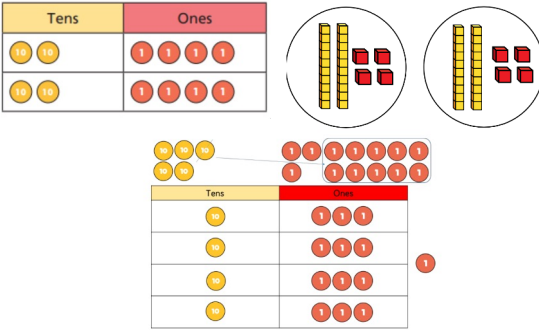
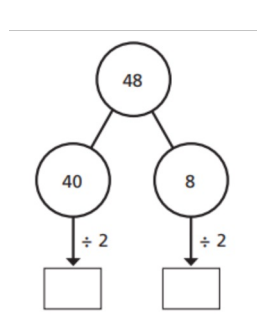
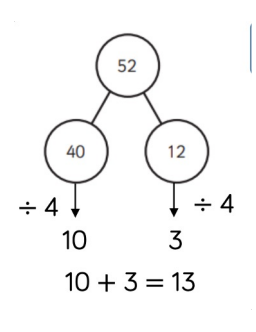
Year 3 - Subtraction

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Subtract numbers mentally including				
> 3-digit number and ones	Autumn Block 2	 $344 - 2 = 342$	$132 - 9 = \square$ 	$315 - 7 = \square$ $\square = 671 - 3$ $18_ - 5 = 178$
> 3-digit number and tens				$146 - 30 = \square$ $612 - \square = 532$ $\square - 40 = 255$ $720 - 60 = \square$
> 3-digit number and hundreds				$561 - 500$  $561 - 300$ How can Jack use this fact to calculate $894 - 500$?
Subtract numbers with up to 3-digits, using formal written methods of columnar subtraction		$254 - 125$ 	Children also draw their own place value charts and record alongside using columnar method before moving to abstract – the should be confident with the principle and recording of exchanging hundreds and tens.	 

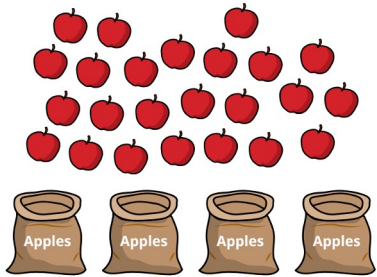
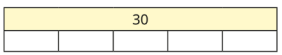
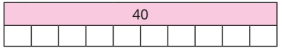
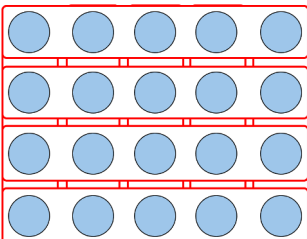
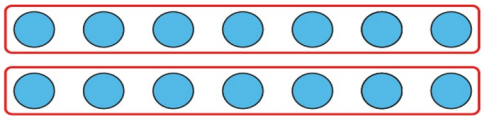
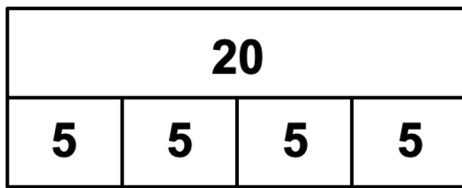
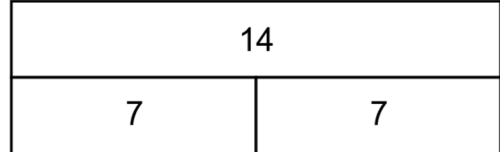
Year 3 - Subtraction

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Solve problems including missing number problems, using number facts, place value and more complex subtraction	Autumn Block 2		Children also draw their own place value charts and record alongside using columnar method before moving to abstract – the should be confident with the principle and recording of exchanging hundreds and tens. 	 
3NF-1 Secure fluency in subtraction facts that bridge 10, through continued practice	Autumn Block 2 (Steps 8, 9, 15 & 16)		 	 
3AS-1 Calculate complements to 100	Summer Block 2 (steps 4 & 5)			£3 and 40p - £1 and 50p = £ and p £8 and 65p - £5 and 65p = £ and p
3AS-2 Subtract up to three-digit numbers using columnar methods	Autumn Block 2 (Steps 12, 15, 16 & 18)		Children draw the place value chart and counters themselves and record alongside using columnar method. 	 
3AS-3 Understand the inverse relationship between addition and subtraction and how both relate to part-part-whole structures	Autumn Block 2 (Steps 21 & 22) Summer Block 2 (Steps 4 & 5)			£3 and 50p - £1 and 50p = £ and p £8 and 65p - £5 and 25p = £ and p

Year 3 - Division

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Recall and use division facts for the 3, 4 and 8 multiplication tables	Autumn Block 3 Spring Block 1	 Complete the division. ÷ =	 ÷ =	$48 \div 2 = 24$
Write ad calculate mathematical statements for division using the multiplications that they know, including two-digit numbers by one-digit using mental methods	Autumn Block 3 Spring Block 1		 	$48 \div 2 = 24$ $52 \div 4 = 13$ $53 \div 4 = 13 \text{ r}1$

Year 3 - Division

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
3MF-2 Recall corresponding division facts in the 10, 5, 2, 4 and 8 multiplication tables	Autumn Block 3 (steps 5 & 10)		$30 \div 5 = \square$  $40 \div 10 = \square$  	$64 \times \underline{\hspace{2cm}} = 640$
3MD1 Apply known division facts to solve contextual problems with different structures, including quotative and partitive division	Autumn Block 3 (all steps) Spring Block 1 (All steps)	 $4 \times 5 = 5 \times 4$  $14 \div 2 = 7$	 $14 \div 2 = 7$ 	$20 \div 5 = 4$

Year 4 - Addition

End of Year 4 Fluency Milestones

Procedural Fluency

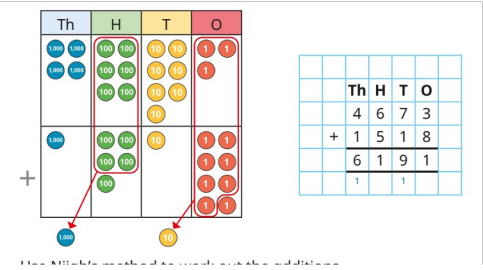
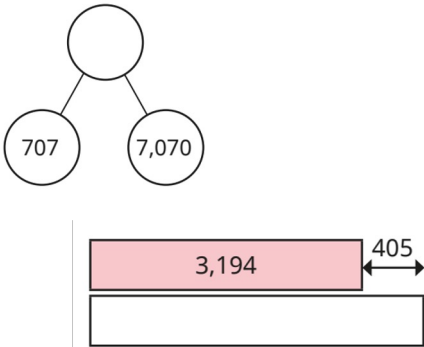
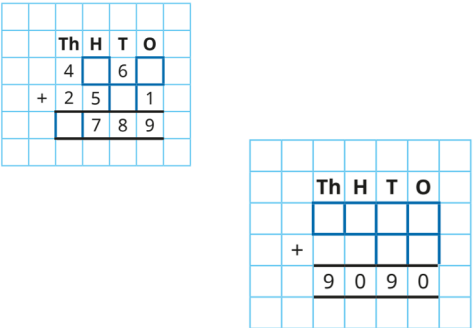
- Apply place value knowledge to known additive and multiplicative number facts (scaling facts by 100) e.g., $8+6=14$ and $14-6=8$ **so** $800+600=1400$ & $1400-600=800$. $3 \times 4=12$ and $12 \div 4=3$ **so** $300 \times 4=1200$ and $1200 \div 4=300$.
- Add and subtract numbers with up to 4-digits using column method, including with exchange accurately.
- Regrouping for addition and subtraction to allow bridging through hundreds, tens and ones – exploring multiple ways of regrouping addend, the subtrahend or the minuend.
- Compensation and rebalance e.g. “*255 + 49 is easier if I take one from the 255 and give it to the 49. My sum stays equal. Then my sum becomes 254 + 50 = 304.*”

Conceptual fluency

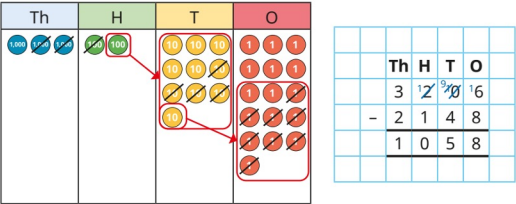
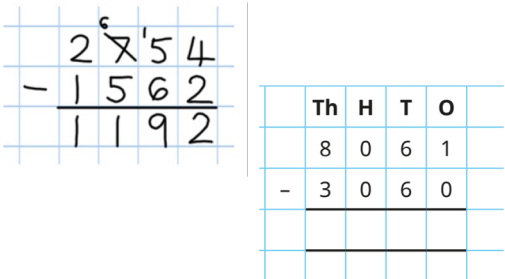
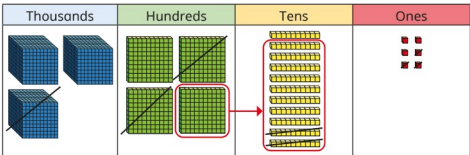
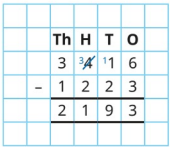
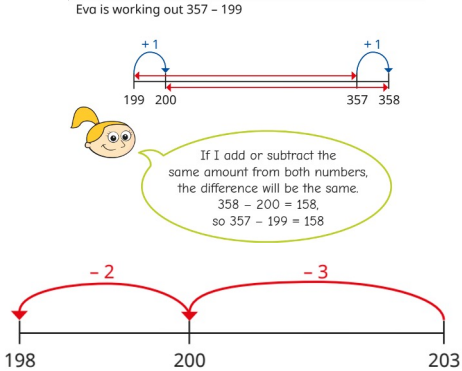
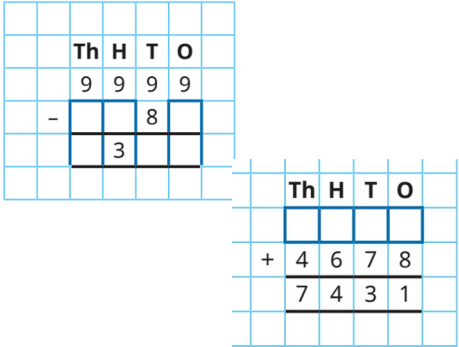
- Rapid recall of multiplication and division facts up to 12×12 , and recognise products in multiplication tables as multiples of the corresponding number.
- Understand the scaling relationship between ones, tens, hundreds and thousands
- Multiply and divide whole numbers by 10 and 100; understanding this as making a number 10 or 100 times the size

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract																									
Add numbers with up to 4-digits using the formal written method of columnar addition where appropriate	Autumn Block 2	314 + 522 =	Draw representations using pv grid.	<table border="1"> <thead> <tr> <th></th> <th>Th</th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td></td> <td>4</td> <td>7</td> <td>1</td> <td>2</td> </tr> <tr> <td>+</td> <td>3</td> <td>4</td> <td>9</td> <td>2</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table> $\begin{array}{r} 2634 \\ + 4517 \\ \hline 7141 \\ \hline 1 \quad 1 \end{array}$		Th	H	T	O		4	7	1	2	+	3	4	9	2										
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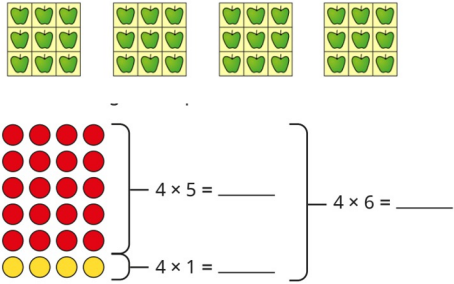
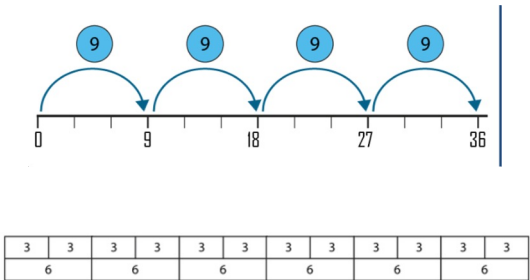
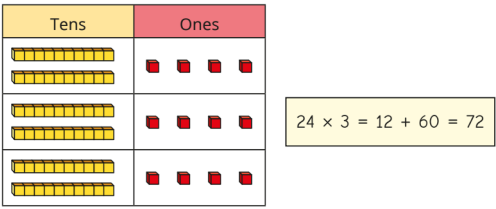
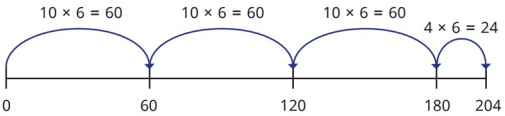
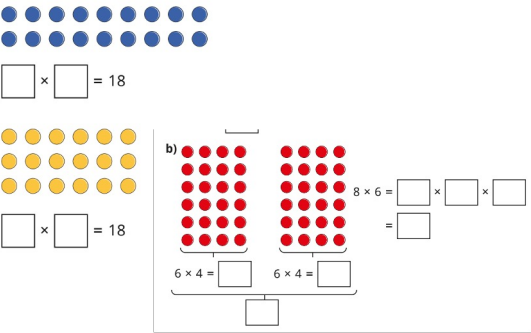
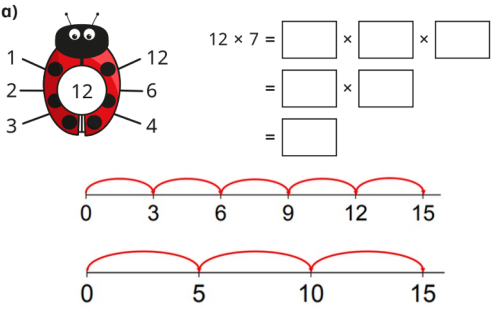
Year 4 - Addition

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Solve addition two-step problems in contexts, deciding which operations and methods to use and why	Autumn Block 2	 The concrete representation shows a place value chart with columns for Thousands (Th), Hundreds (H), Tens (T), and Ones (O). Base ten blocks are used to represent the numbers 1,518 and 1,673. A red arrow indicates the exchange of one hundred block for ten ten blocks. A second place value chart shows the result of the addition: 3,191.	 The pictorial representation shows a tree diagram with a root node and two child nodes labeled 707 and 7,070. Below this, a number line shows the addition of 3,194 and 405, with a double-headed arrow indicating the distance between the two numbers.	 The abstract representation shows two place value charts. The first chart shows the addition of 4,618 and 2,519. The second chart shows the addition of 9,090 and 9,090.

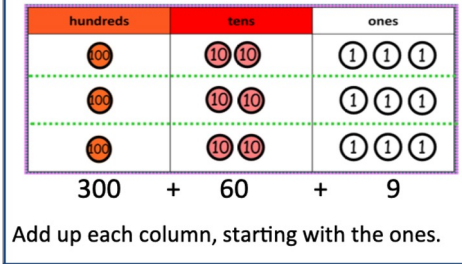
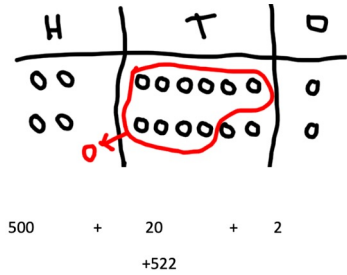
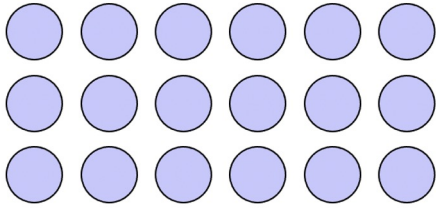
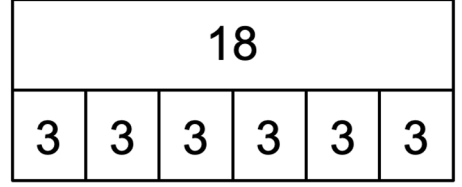
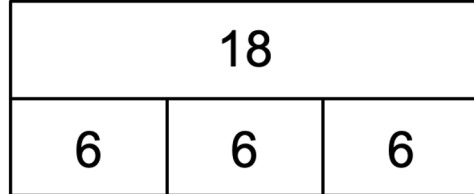
Year 4 - Subtraction

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriate	Autumn Block 2	 Using columnar subtraction to record alongside and support understanding towards abstract methods.	Children also draw their own place value charts and record alongside using columnar method before moving to abstract – the should be confident with the principle and recording of exchanging thousands, hundreds and tens.	
Solve subtraction two-step problems in contexts, deciding which operations and methods to use and why (Including efficient methods)	Autumn Block 2	 	Eva is working out $357 - 199$ 	

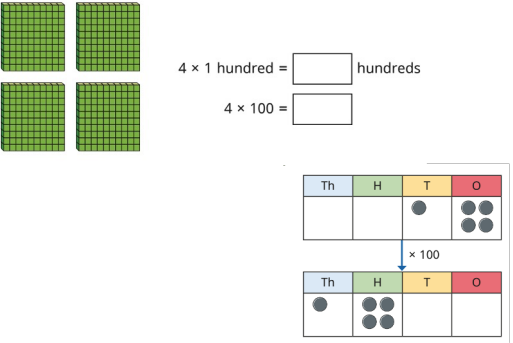
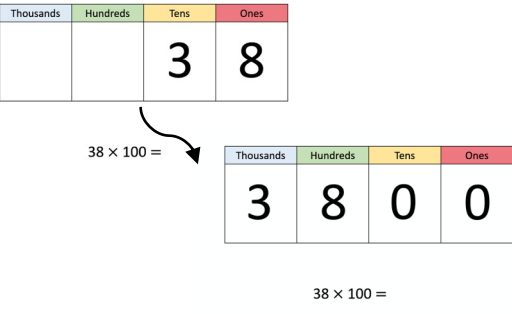
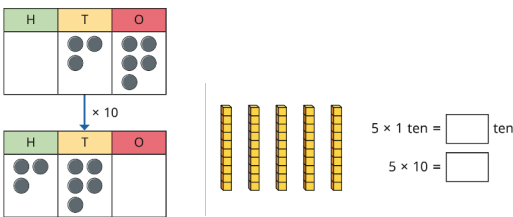
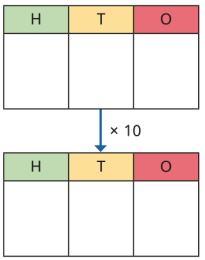
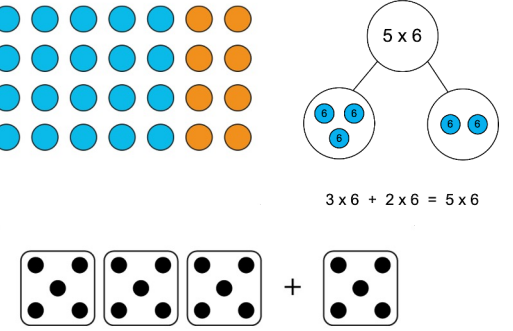
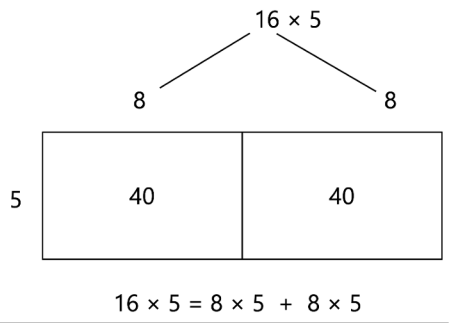
Year 4 - Multiplication

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Recall multiplication facts for multiplication tables up to 12x12	Autumn Block 4 Spring Block 1			There are 36 apples. $4 \times 9 = 36$ $9 \times 4 = 36$
Use place value, known and derived facts to multiply mentally, including by 0 and 1 and multiplying three numbers together	Autumn Block 4 Spring Block 1			$32 \times 10 = 10 \times \square$ $670 = 2 \times 5 \times \square$
Recognise and use factor pairs and commutativity in mental calculations	Autumn Block 4 Spring Block 1			$3 \times 5 = 15$ $5 \times 3 = 15$ $5 \times 3 = 3 \times 5 = 15$

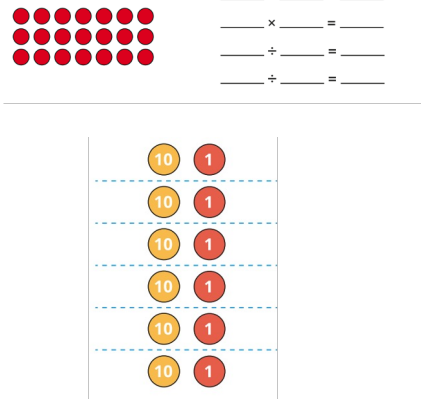
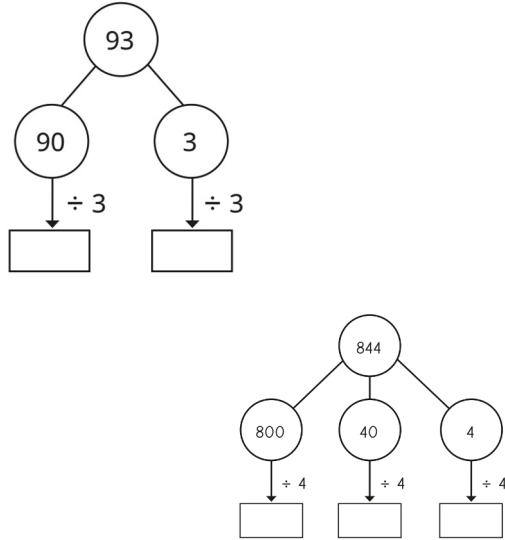
Year 4 - Multiplication

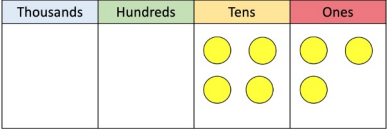
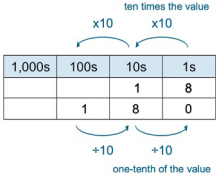
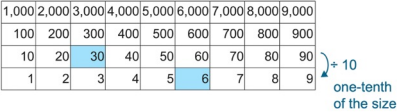
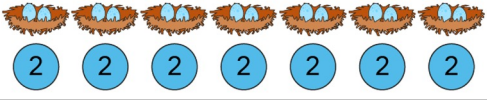
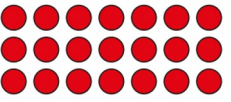
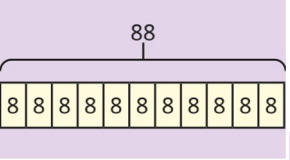
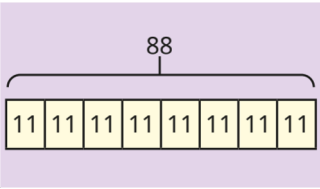
National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout	Spring Block 1	$123 \times 3 = 369$ 		$\begin{array}{r} 241 \\ \times 4 \\ \hline 964 \end{array}$
4NF-1 Recall multiplication facts up to 12×12 and recognise products in multiplication tables such as multiples of the corresponding number.	Autumn Block 4 (All steps) Spring Block 1 (Steps 1, 2, 7, 8, 9 & 10)		 	$6 \times 3 = 18$ $3 \times 6 = 18$

Year 4 - Multiplication

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
4NF-2 Apply place value knowledge to known additive and multiplicative number facts (scaling facts by 100)	Spring Block 1 (Steps 4 & 6) Spring Block 4 (step 10)	 4 × 1 hundred = hundreds 4 × 100 =	 38 × 100 =	$5 \times 7 \times 100 = \boxed{}$ $\boxed{} \times 100 = 6,500$
4MD-1 Multiply whole numbers by 10 and 100 (keeping to whole number quotients) Understand this as equivalent to making a number 10 or 100 times the size	Spring Block 1 (Steps 3 & 4)	 5 × 1 ten = tens 5 × 10 =	Draw counters on the place value charts to show 21 × 10 	$10 \times \boxed{} = 140$ $\boxed{} = 40 \times 10$ $32 \times 10 = 10 \times \boxed{}$
4MD-3 Understand and apply the distributive property of multiplication	Spring Block 1 (Steps 8, 9 & 10)	 3 × 6 + 2 × 6 = 5 × 6	 16 × 5 = 8 × 5 + 8 × 5	$7 \times 4 = 5 \times 4 + 2 \times 4$

Year 4 - Division

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Recall division facts fie multiplication tables up to 12x12	Autumn Block 4 Spring Block 1	 _____ × _____ = _____ _____ × _____ = _____ _____ ÷ _____ = _____ _____ ÷ _____ = _____		108 ÷ 9 = 12 81 ÷ 9 = 9 27 ÷ 9 = 3
4NF-2 Solve division problems, with 2-digit dividends and 1-digit divisors, that involve remainders, and interpret remainders appropriately according to the context	Autumn Block 4 (All steps) Spring Block 1 (steps 11, 12 & 13)			648 ÷ 4 = 847 ÷ 7 =

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
4MD-1 Divide whole numbers by 10 and 100 (keeping to whole numbers)	Spring Block 1 (Step[s 5 & 6])	 $4,300 \div 100 = 43$	Using place value and gattegno charts $180 \div 10$  $36 \times 10 = 360$ $360 \div 10 = 36$ 	Knowing related calculations $570 \div 10$ $9,500 \div 100 = 95$
4MD-2 Manipulate division equations	Autumn Block 4 (All steps)	  _____ ÷ _____ = _____ _____ ÷ _____ = _____	 	$14 \div 7 = 2$ $14 \div 2 = 7$

Exchange the divisor and quotient and the dividend remains the same.

Year 4 - Division

Year 5 - Addition

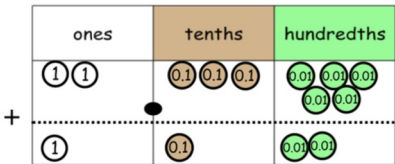
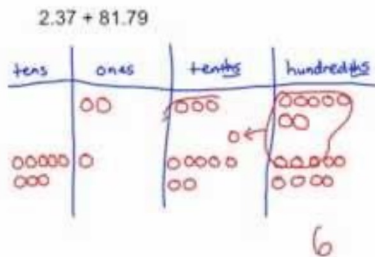
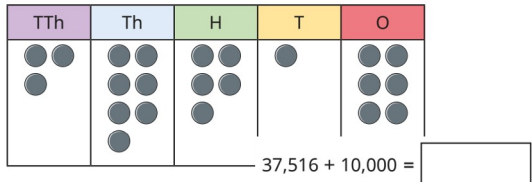
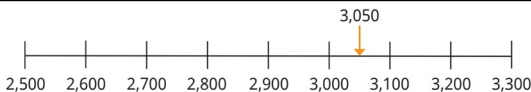
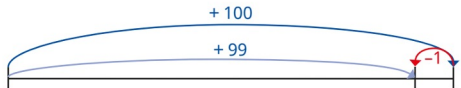
End of Year 5 Fluency Milestones

Procedural Fluency

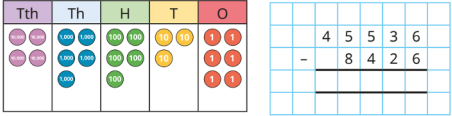
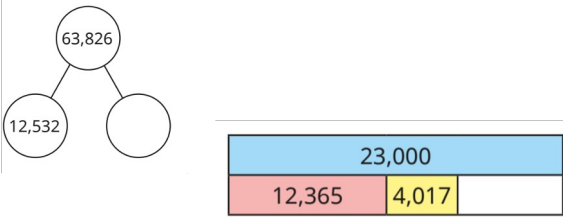
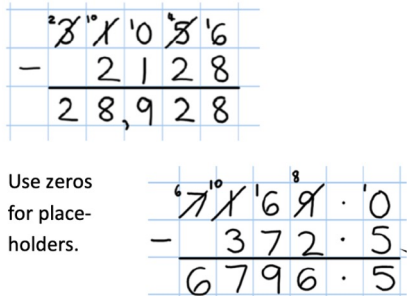
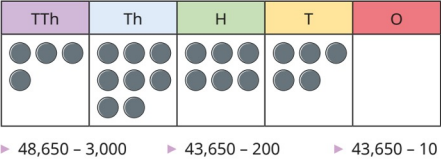
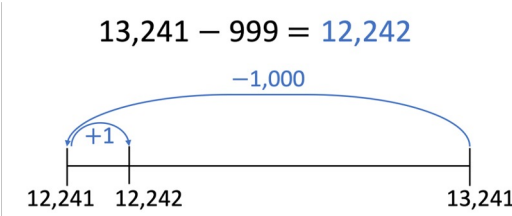
- Apply place-value knowledge to known additive and multiplicative number facts (scaling facts by 1 tenth or 1 hundredth), for example:
 $8+6=14$. $0.8+0.6=1.4$. $0.08+0.06=0.14$
- Scale additive calculations related to complements to 100 e.g. $62 + 38 = 100$ so $0.62 + 0.38 = 1$
- Formal column methods for addition and subtraction
- Formal short multiplication and division.
- Regrouping for addition and subtraction to allow bridging through hundreds, tens and ones – exploring multiple ways of regrouping. Then adapted to decimal and fractional part whole as well as measures such as time and money.

Conceptual fluency

- Secure fluency in multiplication table facts to 12×12 , and corresponding division facts, through continued practice.
- Multiply and divide numbers by 10 and 100; understand this as equivalent to making a number 10 or 100 times the size, or 1 tenth or 1 hundredth times the size.
- Know factors and multiples of positive whole numbers, including common factors and common multiples.

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Add whole numbers with more than 4 digits, including using formal written methods (column)	Autumn Block 2	As year 4  Introduce decimal place value counters		$\begin{array}{r} 22,634 \\ + 15,673 \\ \hline 38,307 \end{array}$ $\begin{array}{r} \text{£ } 127.67 \\ + \text{£ } 38.45 \\ \hline \text{£ } 166.12 \end{array}$
Add numbers mentally with increasingly large numbers	Autumn Block 2	 $37,516 + 10,000 = \boxed{}$ $37,516 + 1,000 = \boxed{}$	 a) $3,050 + 100 = \boxed{}$ c) $3,050 + 200 = \boxed{}$ 	$2,360 + 99 = \boxed{}$ $52,871 + 99 = \boxed{}$

Year 5 - Subtraction

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
Subtract whole numbers with more than 4 digits including formal written methods (columnar)	Autumn Block 2	Using column method to record 	Children draw place value counters to show their exchange – using column method to record. 	 Use zeros for place-holders.
Subtract numbers mentally with increasingly large numbers	Autumn 2	 ▶ 48,650 – 3,000 ▶ 43,650 – 200 ▶ 43,650 – 10	$13,241 - 999 = 12,242$ 	$90,070 - 30,060 = 60,010$ I can partition 90,070 into <u>90,000</u> and <u>70</u> and partition 30,060 into <u>30,000</u> and <u>60</u> and then subtract the parts separately. $90,000 - 30,000 = 60,000$ $70 - 60 = 10$ <u>$4,150 - 300 = 3,850$</u>

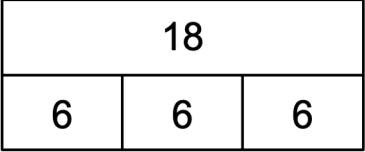
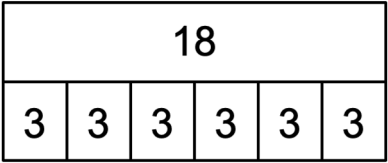
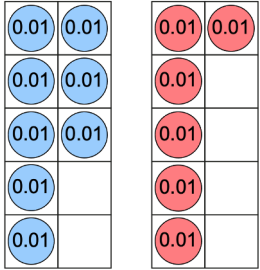
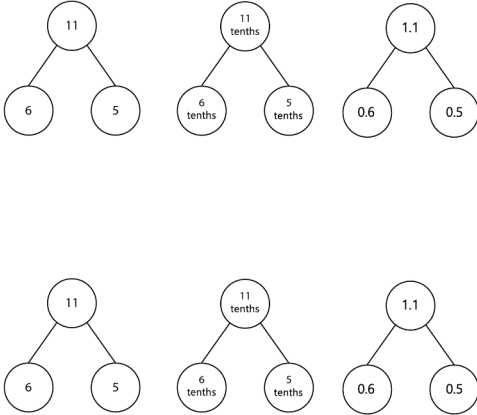
Year 5 - Multiplication

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract																																																																																																		
Identify multiples and factors including finding all factor pairs of a number, and common factors of two numbers	Autumn Block 3	Both numbers can be arranged in one row, so 1 is a common factor: 12 can be arranged in two rows but 15 cannot, so 2 is not a common factor: <table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr><tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr><tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr><tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr><tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr><tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr><tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr><tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr></table> Shade the first ten multiples of 5 Circle the first ten multiples of 3 List the first two common multiples of 5 and 3 What is the next common multiple of 5 and 3? Find some more common multiples of 5 and 3 12 18 24 9 6 45 48 54 36 63 multiples of 6 multiples of 9 2 × _____ = 14 so 6 × 14 = 6 × 2 × _____ 3 × _____ = 9 so 9 × 12 = 3 × _____ × 12	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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91	92	93	94	95	96	97	98	99	100																																																																																													
Establish whether a number up to 100 is prime and recall prime numbers up to 19	Autumn Block 3		<table><tr><th>Number</th><th>Factors</th><th>Is the number prime?</th></tr><tr><td>5</td><td>1 and 5</td><td>Yes</td></tr><tr><td>9</td><td></td><td></td></tr><tr><td>11</td><td></td><td></td></tr><tr><td>14</td><td></td><td></td></tr><tr><td>15</td><td></td><td></td></tr><tr><td>19</td><td></td><td></td></tr></table>	Number	Factors	Is the number prime?	5	1 and 5	Yes	9			11			14			15			19			Here are sequences of consecutive prime numbers. Complete the sequences. a) 7, 11, 13, , 19 b) 37, 31, 29, , 19																																																																													
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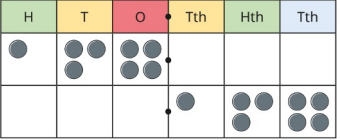
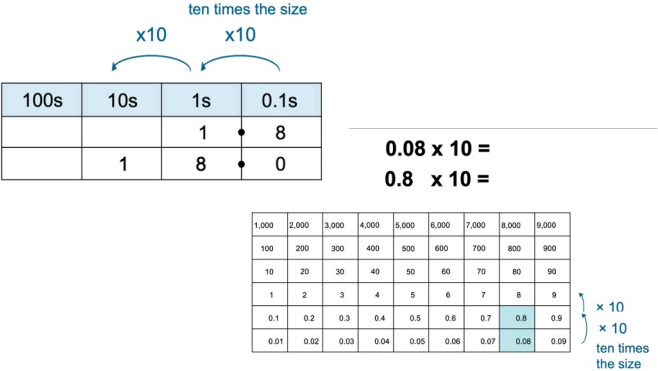
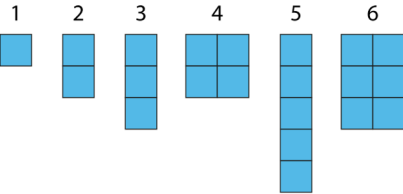
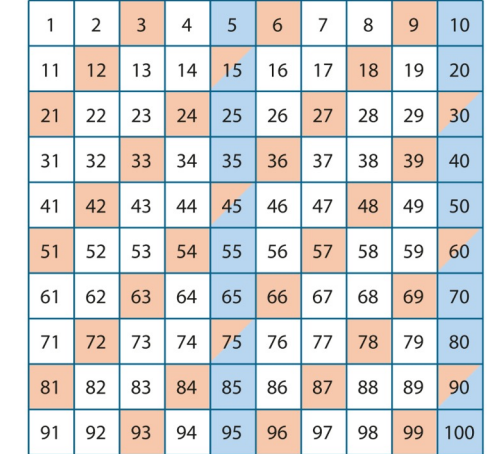
Year 5 - Multiplication

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract																																																						
Recognise and use square numbers and cube numbers, and for the notation for square and cubed	Autumn Block 3	● ●● ●●● ●●●● $1 \times 1 = \square$$2 \times 2 = \square$$3 \times \square = \square$$\square \times \square = \square$		a) $6^2 = \square$ b) $12^2 = \square$ c) $\square = 9^2$ d) $0^2 = \square$ e) $\square^2 = 100$ f) $64 = \square^2$ $6^3 = (6 \times 6) \times 6$ $= 36 \times 6$ $= 216$ $30 \quad 6$ $6 \begin{array}{ c c } \hline 30 \times 6 = 180 & 6 \times 6 = 36 \\ \hline \end{array}$ $180 + 36 = 216$																																																						
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method including long multiplication for two-digit numbers	Autumn Block 3 Spring Block 1	<table><tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td>1000</td><td>100</td><td>10</td><td>1</td></tr><tr><td>1000</td><td>100</td><td>10</td><td>1</td></tr><tr><td>1000</td><td>100</td><td>10</td><td>1</td></tr><tr><td>1000</td><td>100</td><td>10</td><td>1</td></tr></table> 2 2 1 4 × 4 3024 × 3 <table><tr><th>Thousands</th><th>Hundreds</th><th>Tens</th><th>Ones</th></tr><tr><td>3000</td><td></td><td>20</td><td>4</td></tr><tr><td>3000</td><td></td><td>20</td><td>4</td></tr><tr><td>3000</td><td></td><td>20</td><td>4</td></tr></table> 900 + 0 + 60 +	Th	H	T	O	1000	100	10	1	1000	100	10	1	1000	100	10	1	1000	100	10	1	Thousands	Hundreds	Tens	Ones	3000		20	4	3000		20	4	3000		20	4	Children may continue to draw their understanding using place value grids.	<table><tr><td></td><td>1</td><td>8</td></tr><tr><td>×</td><td>1</td><td>3</td></tr><tr><td></td><td>5</td><td>4</td></tr><tr><td></td><td>2</td><td></td></tr><tr><td>1</td><td>8</td><td>0</td></tr><tr><td>2</td><td>3</td><td>4</td></tr></table> 3024 x 3 9072 1		1	8	×	1	3		5	4		2		1	8	0	2	3	4
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Multiply whole numbers and those involving decimals by 10, 100 and 1000	Autumn Block 3 Spring Block 1	<table><tr><th>HTh</th><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td></td><td></td><td>2</td><td>3</td><td>4</td></tr></table> If you multiply 234 by 10, where do the counters move to? What is the result of multiplying 234 by 10? If you multiply 234 by 100, where do the counters move to? What is the result of multiplying 234 by 100?	HTh	TTh	Th	H	T	O				2	3	4	5,100 5,000 100 35,000 700 <table><tr><th>HTh</th><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td></td><td></td><td>2</td><td>4</td><td>5</td></tr></table> 245 × 100 = 245 × 1,000 =	HTh	TTh	Th	H	T	O				2	4	5	45 × 100 = 52 × 100 = 70 × 100 = c) 41 × 10 = 41 × 100 = 41 × 1,000 =																														
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HTh	TTh	Th	H	T	O																																																					
			2	4	5																																																					

Year 5 - Multiplication

National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
SNF-1 Secure fluency in multiplication table facts, through continued practice	Autumn Block 3 (Steps 1, 2, 3, 4 & 6) Spring Block 1 (All steps) Spring Block 2 (All steps)		 	$6 \times 3 = 18$ $3 \times 6 = 18$ $\frac{1}{6} \text{ of } 18 = 3$ $\frac{1}{3} \text{ of } 18 = 6$
SNF-2 Apply place value knowledge to known multiplicative number facts (scaling facts by 1 tenth or 1 hundredth)	Autumn Block 3 (Step 10)	 $8 + 6$ $8 \text{ hundredths} + 6 \text{ hundredths}$ $0.08 + 0.06$		$4 \times 2 = 8$ $4 \times 0.2 =$ $12 \times 4 = 48$ $12 \times 0.4 =$

Year 5 - Multiplication

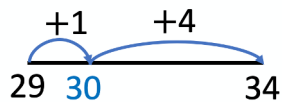
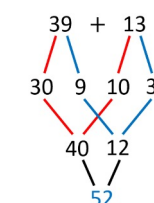
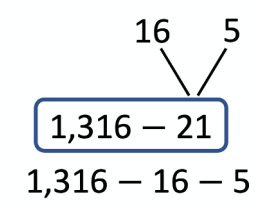
National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
5MD-1 Multiply numbers by 10 and 100; understand this as equivalent to making a number 10 or 100 times the size, or 1 tenth or 1 hundredth times the size	Autumn Block 3 (Steps 8, 9 & 10) Summer Block 3 (Steps 10, 11 & 12)	 The counters have moved places to the The number has been by	Using place value and gattegno charts.  $0.08 \times 10 = 0.8$	$4.7 \times 10 =$ $4.7 \times 100 =$ $4.7 \times 1,000 =$ 1.32 m = cm £31.76 = p 0.458 kg = g 1.302 l = ml
5MD-2 Find factors and multiples of positive whole numbers, including common factors and common multiples, and express a given number as a product of 2 or 3 factors	Autumn Block 3 (Steps 1, 2, 3, 4 and 6)			List the first three common multiples of each pair of numbers. a) 3 and 5 b) 2 and 7 c) 9 and 6

Year 5 - Division

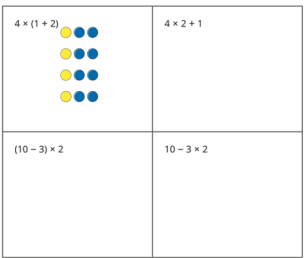
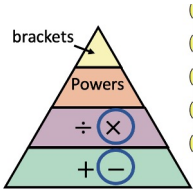
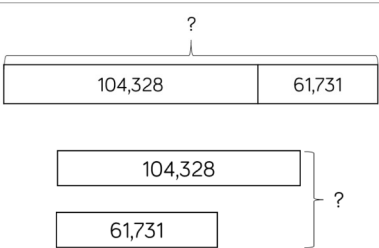
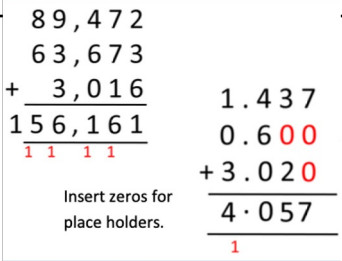
National Curriculum objective or RTP criteria	When taught?	Concrete	Pictorial	Abstract
5MD-4 Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	Autumn Block 3 Spring Block 1 (Steps 7, 8 & 9)		Children draw their own pictorial representations and record alongside using formal method. $6,078 \div 6 = \square$	
Divide whole numbers and those involving decimals by 10, 100 and 1000	Autumn Block 3 Spring Block 1	If you divide the number by 10, where do the counters move to? What is the result of dividing the number by 10? If you divide the number by 100, where do the counters move to? What is the result of dividing the number by 100?	$60 \div 10 = \square$ $16,000 \div 100 = \square$ $768,000 \div 1,000 = \square$	$4,500 \div 100 = \square$ $62,000 \div 100 = \square$ $739,300 \div 100 = \square$ $\square \div 1,000 = 30$ $\square \div 1,000 = 300$ $\square \div 1,000 = 3,000$ © White Rose Maths 2022

Year 6 - Addition

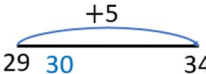
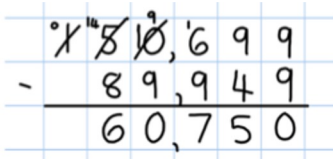
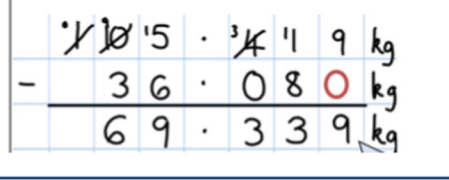
End of Year 6 Fluency Milestones	
<u>Procedural Fluency</u> - Multiply multi digit numbers up to 4-digits by a 2-digit whole number using formal written long multiplication - Divide numbers up to 4 digits by a two-digit whole number using the formal written method of short and long division, - Continue to develop fluency in regrouping for addition and subtraction to allow bridging through hundreds, tens and ones – exploring multiple ways of regrouping. Then adapted to decimal and fractional part whole as well as measures such as time and money. - Compensation with the same calculation supports pupil's multi-strategy approach. Pupils can continue to evaluate strategies e.g., $7834 + 79,996$ "Adding 79,996 is like adding 80,000 and subtracting 4. I can do $80,000 + 7834 - 4 = 87,830$" - Equal difference using comparison drawing out the concept that adding or subtracting the same quantity from both the subtrahend and minuend will maintain the difference between the numbers. E.g., $132,457 - 11,999 =$ "11,999 is nearly 12,000. If I add one to each number the difference will stay equal. Now my calculation is $132,458 - 12,000 =$"	<u>Conceptual fluency</u> Continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency

National Curriculum objective or RTP criteria	When taught?	Pictorial	Abstract
Perform mental calculations with large numbers	Autumn Block 2	<u>Strategy 1 – Count on</u> 	<u>Strategy 1 – Partitioning and adding</u>  <u>Strategy 2 – Round then add</u> $\begin{aligned} &39 + 13 \\ &+1 \quad \curvearrowright \quad 40 + 13 = 53 \\ &\quad \quad \quad 53 - 1 = 52 \end{aligned}$ <u>Strategy 2 – Constant difference</u> $\begin{aligned} &+1 \quad \left(\begin{array}{l} 34 - 29 \\ 35 - 30 \end{array} \right) +1 \\ &\quad \quad \quad 35 - 30 \end{aligned}$ 

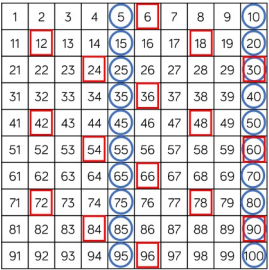
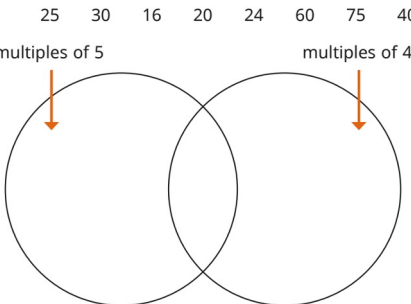
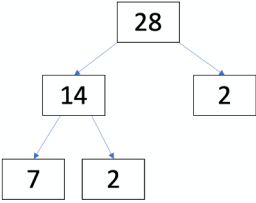
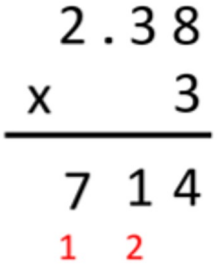
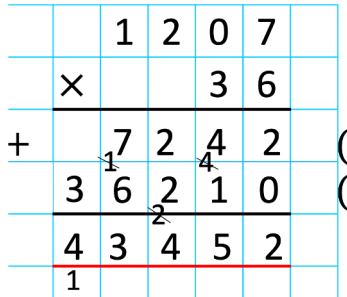
Year 6 - Addition

National Curriculum objective or RTP criteria	When taught?	Pictorial	Abstract
Use knowledge of the order of operations to carry out calculations	Autumn Block 2	Draw representations to match calculations. 	Annie has 35 stickers. She gives <u>3</u> of her friends 7 stickers each. $35 - 3 \times 7 = 14$ 
Solve addition multi step problems in contexts, deciding which methods to use and why.	Autumn 2		
6AS/MD -2 Use given additive calculations to derive or complete a related calculation , using arithmetic properties, inverse relationship and place value understanding	Autumn 2		Question: Explain how you would use the first equation to complete the second equation: $921 = 349 + 572$ $92.1 = 44.9 + \square$ 1. Apply understanding of place value, making the sum and addends 1 tenth times the size. $92.1 = 34.9 + 57.2$ 2. Apply the compensation property of addition to solve the equation: add 10 to the first addend and subtract 10 from the second addend. $92.1 = 44.9 + 47.2$ $327 + 278 = 330 + \square$ $327 + 515 = 842$ Use this calculation to complete the following equations. $\square + 61.5 = 84.2$ $8,420 - \square = 3,270$ $85,200 - 52,500 = \square$

Year 6 - Subtraction

National Curriculum objective or RTP criteria	When taught?	Pictorial	Abstract
Perform mental calculations with large numbers	Autumn Block 2		Count on  Constant difference $+1 \begin{pmatrix} 34 - 29 \\ 35 - 30 \end{pmatrix} +1$ $35 - 30$ $101 - 36$ $\begin{matrix} -1 & & -1 \\ \curvearrowleft & & \curvearrowright \end{matrix}$ $100 - 35$ $100 - 35 = 65$ $3,001 - 2,999$ $2,999 + 2 = 3,001$ $1,316 - 21$ $1,316 - 16 - 5$ $1,300 - 5 = 1,295$
Solve subtraction multi step problems in contexts, deciding which methods to use and why.	Autumn 2		 

Year 6 - Multiplication

National Curriculum objective or RTP criteria	When taught?	Pictorial	Abstract
Identify common factors, common multiples and prime numbers	Autumn Block 2	 Find all the common multiples of 5 and 6	Find all the prime factors of 28   $7 \times 2 \times 2 = 28$
Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication	Autumn Block 2		Children should be confident in using the formal written method. If they are still struggling, provide multiplication grids to support when they are focusing on the use of the method, Consider where exchanged digits are places and make sure this is consistent.   $(1,207 \times 6)$ $(1,207 \times 30)$

National Curriculum objective or RTP criteria	When taught?	Pictorial	Abstract
6MD-2 Use a given multiplicative calculation to derive or complete a related calculation using arithmetic properties, inverse relationships and place value understanding	Autumn Block 2 (Steps 8, 14 & 17)	A machine can pack 1,010 boxes each day. There are 32 machines in a factory. How many boxes can be packed in one day? 32 × 1,000 × 10	17 × 8 = 170 × 7 = 170 × 70 = 17 × 7 = 119 119 ÷ 7 = 117 × 7 = 170 × = 11.9 1,190 ÷ = 70 4,625 + 3,709 = 8,334 Use this fact to complete the calculations. 3,709 + 4,625 = 8,334 − 3,709 = 46,250 + 37,090 = d) 4,625 + 3,709 = 4,630 + e) 4,625 + 3,709 = + 3,700 f) 8,334 − 3,709 = − 3,700

Year 6 - Multiplication

Year 6 - Division

National Curriculum objective or RTP criteria	When taught?	Abstract
Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context	Autumn Block 2	H T O 31 4 3 4 → H T O 0 1 31 4 3 4 → H T O 0 1 31 4 3 4 → H T O 0 1 31 4 3 4 31 does not go into 4 (hundreds). We combine the 4 hundreds with the tens to give 43 tens. 31 goes into 43 once which is 31, we record this underneath. Subtract to find the remainder. 31 from 43 leaves 12. H T O 0 1 31 4 3 4 → H T O 0 1 31 4 3 4 → H T O 0 1 31 4 3 4 We combine 12 with the next digit to give 124. 31 goes into 124 four times, which is 124. We subtract to show there is no remainder 1. Divide. t o 2 2 5 8 Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder! 2. Multiply & subtract. t o 2 2 5 8 -4 1 To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten. 3. Drop down the next digit. t o 2 9 2 5 8 -4 1 8 Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18. 1. Divide. t o 2 9 2 5 8 -4 1 8 Divide 2 into 18. Place 9 into the quotient. 2. Multiply & subtract. t o 2 9 2 5 8 -4 1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract. 3. Drop down the next digit. t o 2 9 2 5 8 -4 1 8 0 The division is over since there are no more digits in the dividend. The quotient is 29.
Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context	Autumn Block 2	Written methods become the most accurate as concrete and pictorial become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient (the result of a division) can be rounded as appropriate. 15 30 45 60 75 90 105 120 135 150 0 4 1 6 2 4 6 0 3 6 12 4 43 72